Energy, Technology, and Economic Growth

Understanding the implications of energy constraints on economic growth and the energy transition

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“All natural processes and all human actions are, in the most fundamental physical sense, transformations of energy.”

- Vaclav Smil

Foreword

It is hard to describe the milestone that this moment represents in my life. For me, this is not just my master's thesis. It is a point of reflection on how I got here. The last year and a half came with some unprecedented challenges in my personal life. Everything I thought I knew - about myself, about the world, about the future - was turned upside down.

As I lost all sense of direction, I considered quitting university altogether. But after taking a good 7 months to myself, resting and recovering, I grabbed a piece of paper and I wrote: What are my interests and drives?

I made a list of things that I felt passionate learning more about. This included ecological macroeconomics, material use, modeling post-growth scenarios, and learning Python. It is crazy to look back at this and to realize how much I have learned in the last year, both inside and outside of university. Everything felt so uncertain, but I now know that I made it. I even enjoyed the process. Everyone told me that after 6 months I would be tired of this topic, but I am more excited than ever before.

There are so many people to thank for supporting me throughout this time, from my friends from home to people I briefly crossed paths with while I was abroad. I specifically want to thank Franziska and Théo for giving me a soft landing back in Wageningen, for the endless meetings and emails past work hours, and for getting me my dream internship in Barcelona. Ignacio, for being the best friend I could wish for, especially by lending me the Vaclav Smil book that became a huge theoretical backbone for this thesis (you will get it back soon, I promise). Emiliano, for always making me feel heard and supported, and for giving me confidence when I doubted myself. And lastly, to my family. To my mom and dad, thank you for never judging me for taking my time. And to my sisters, Eva, Julia, and Sara, for always being the home I return to when I am lost. I am indebted to all of you.

Abstract

Standard economic theory suggests that technological progress is key in achieving long-run economic growth, but this is being challenged by ecological economists who emphasize the central role of energy in production. Historically, many innovations that led to growth were simultaneously linked to the increased use of fossil fuels, leading to a host of environmental problems. Through further innovations, the energy transition is now supposed to delink economic growth from its ecological impacts, but this comes with a large mineral footprint which is often overlooked in environmental-economic models. These models mainly focus on decarbonization without looking at the biophysical basis of our economy. This thesis therefore argues for a different conceptualization of energy in economic thinking, in line with physical laws. To analyze the impact of making the role of energy in production nontrivial, important changes are made to an existing modeling framework of the energy transition by Golosov et al. (2014). Specifically, I change the formulation of the Cobb-Douglas production function, and integrate mineral constraints to the production of low-carbon energy. Applying these changes, and calibrating the model to the use of copper in the energy transition, the model finds that mineral constraints lead to a plateau in low-carbon energy production, the timing and height of which are determined by technological assumptions. Whether sustainable growth is possible then depends on the tradeoff between technological innovation that is (i) energy-saving, reducing the energy intensity per unit of output and offsetting the impact of scarcity, or (ii) energy-increasing, by expanding capital's energy throughput. Overall, the findings suggest that the trajectory of economic growth is ultimately determined by the complementary relationship between the aggregate usable energy supply and the ability of capital to harness that energy.

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1 Introduction

There is an urgency to find ways to meet human needs while staying within planetary boundaries (Steffen et al., 2015). Among these boundaries, climate change has emerged as one of the most pressing issues. By 2050, most nations have therefore pledged to become carbon neutral, and one of the key strategies to achieve this is through the energy transition (IEA, 2021). By moving away from a fossil-fuel based society to one based on low-carbon energy, the aim is to reduce greenhouse gas emissions to levels that can keep global warming under 1.5°C. Innovation lies at the heart of achieving these climate ambitions, as many technologies that the success of decarbonization depends upon, such as carbon capture and storage (CCS) or green hydrogen electrolysis, are not yet possible or applicable at scale. Investing in low-carbon infrastructure and R&D is therefore a top priority for policymakers. Yet focusing on emissions alone risks overlooking other sustainability challenges, such as critical mineral scarcity.

The energy transition requires technologies that depend heavily on mineral inputs. Most low-carbon technologies are more mineral-intensive than their fossil-fuel counterparts. Electric vehicles, for instance, require six times the mineral inputs of conventional cars, and onshore wind farms require nine times more mineral resources than gas-fired power plants of the same capacity (Kalt et al., 2022). Worldwide, if countries meet their Paris Agreement goals, this could lead to a quadrupling of the total mineral demand for clean energy technologies (IEA, 2024a). For certain minerals, the increase in consumption is projected to lead to a gap between supply and demand in as little as 10 years (IEA, 2025). There seems to be a trade-off between reducing emissions and increasing mineral scarcity. The transition to a low-carbon economy could thus mean a shift from a fuel-intensive to a material-intensive energy system.

In standard economic theory, innovation and substitution are key for solving challenges around scarcity and a lack of available technologies. The depletion of a non-renewable resource, such as critical minerals, will over time lead to higher prices, which in turn incentivizes producers to increase efficiency or to switch to alternative inputs (Perman et al., 2011). Recent successes like solar energy overtaking coal as the main source of electricity worldwide could support the notion that innovation and substitution can reduce environmental impacts while maintaining a positive level of economic growth. Through this process of technological progress and creative destruction, innovation simultaneously becomes the main driver of long-run economic growth (Aghion & Howitt, 1992).

In standard environmental-economic models, that aim to answer if and how economic growth can be sustained under increasing ecological impacts, innovation therefore also plays an important role. These models traditionally adopt a Cobb-Douglas production function, where innovation, captured by the term Total Factor Productivity (TFP), is seen as the main driver of economic growth. TFP is often an unexplained residual, yet it is used to explain why past resource constraints were overcome. In a nutshell, the idea is that as human ingenuity is endless, so is innovation, and therefore economic growth (Susskind, 2024).

Ecological economists challenge this standard account of economic growth, by highlighting the physical basis of our economy and the importance of energy in production. They posit that historically, the contribution of energy to output has been underestimated due to the specification of the Cobb-Douglas production function, which attaches weights to inputs based on their cost shares (Keen et al., 2019). Through this, energy is treated as a negligible or substitutable input, alongside capital and labour. Since energy was abundantly available, its price remained low relative to other inputs despite its large environmental impact. As a consequence, the contribution of energy to output may have been underestimated, and "unexplained" growth that was captured by TFP, may have been driven by biophysical increases in the amount of energy used in production. This challenges the belief that ever-growing TFP can mitigate the impacts of resource constraints (Flament, 2023; Giraud & Kahraman, 2014).

In the context of the energy transition, innovation and substitution might not be enough to reduce the environmental impacts associated with increased energy use. Despite reducing the emission intensity of certain economies, past innovations and substitutions have not led to a reduction of our worldwide energy requirements (Vogel & Hickel, 2023). Sustainability challenges as well as sustainable technologies are becoming increasingly complex. Replacing one natural resource with the other, like wood with coal, is not comparable to the switch from gas 1 to green hydrogen. Gains from circularity and energy efficiency in high-emitting sectors like steel and cement are not expected to outgrow the environmental pressures related to the drastic increase in mineral demand (van Ruijven et al., 2016). There are limits to substitution, efficiency, and circularity. This emphasizes the importance of taking energy seriously when modeling the complex interplay between energy, technology, and economic growth.

In sum, the energy transition heavily depends on the input of increasingly scarce mineral resources. This reliance on mineral inputs is expected to lead to a gap in supply and demand in the short term, and potential depletion in the long term. Nevertheless, standard environmental-economic models rarely take into account biophysical constraints such as mineral scarcity, because economic theory posits that scarcity can be overcome through innovation and substitution. The specification of the commonly used Cobb-Douglas production function implies that energy has only a trivial role in explaining past and future economic growth. The overall lack of attention to the link between energy and the economy broadly, and to constraints to low-carbon energy specifically, is limiting our understanding of the true economic impact of the energy transition. It is therefore necessary to reconceptualize the role of energy in production, to better capture the interdependency between the economy and the environment. To address these concerns, this thesis aims to answer the following research question:

What are the impacts of energy constraints on the energy transition and economic growth?

To answer the research question, this thesis changes the role of energy in an existing simple IAM created by Golosov et al. (2014) to illustrate the impact of the energy transition on economic outcomes. Specifically, it does so through two channels. First, by changing the production function from a "standard" Cobb-Douglas function to an energy-based production function. This idea is largely in line with the work from Keen et al. (2019). In this new function, the role of energy is nontrivial as capital and energy are perfect complements, and the function is expressed in units of energy. The specification of this function implies that energy can no longer be substituted for by capital for a given technology, and that energy is thus a prerequisite for economic activity and economic growth. In short, without energy there is no production.

The second channel that changes the relationship between energy and the economy is the inclusion of mineral constraints in low-carbon energy production. Linking the production of low-carbon energy to the depletion of mineral reserves better reflects the material basis of our energy system. It challenges the idea that low-carbon energy is infinitely available. The expected outcome is therefore also twofold. Mineral scarcity presents a limit to the scale and development of the low-carbon energy sector, constraining growth of the energy supply. If aggregate economic growth depends on energy, and energy production is limited by increasing scarcity, growth will likely reach its limits, the timing and extent of which will depend on assumptions around technological innovation.

This study thus contributes to the literature methodologically, by changing the Cobb-Douglas production function, and conceptually, by engaging with a growing share of ecological economic literature that address the link between energy use and economic growth, notably Ayres and Warr (2005), Georgescu-Roegen (1979), Hall and Klitgaard (2012), Hickel et al. (2021), and Stern (2011). Through this, it aims to address the gap between standard economic modeling practices and ecological economic ideas on the link between the economy and biophysical resource use. It addresses the key debate of whether economic growth is inherently constrained by resource scarcity, or if innovation allows us to outgrow the curse of diminishing returns.

Serving as the benchmark for this paper is the work by Golosov et al. (2014) and the extension of their work by Chazel et al. (2023). Chazel et al. (2023) adapt Golosov's GHKT model and introduce primary mineral constraints and their potential recyclability. To the best of my knowledge, this is currently the only general equilibrium model that addresses the issue of critical mineral constraints in the energy transition, which is why it was chosen for this thesis. They calibrate their model to the case of copper, and find that even in the hypothetical case of a 100% recycling rate, low-carbon energy production will still plateau in the coming century. This aligns with the findings of Fabre et al. (2020) and Pommeret et al. (2022) who also conclude that including scarcity of critical minerals calls for stronger and earlier investments in renewable energy and that recycling reduces the costs of the transition. These results highlight the importance of taking resource constraints seriously when analyzing the impact of climate policies.

At the same time, as the main change to the model will be the new production function, I depart from Chazel et al. (2023) in several important ways. My aim is not to compute the optimal carbon tax, or to model the impacts of recycling, but to illustrate the change in outcomes when energy is treated as a prerequisite to the production process. This goes beyond modeling the exact paths and peaks of resource extraction, and should be seen as a more conceptual change.

This thesis will proceed as follows. In Section 2, a technical background is provided on critical minerals in the energy transition. This section illustrates the importance of minerals and their respective bottleneck issues, and touches on their projected scarcities. The aim of the theoretical framework in Section 3 is to provide an overview of an extensive body of environmental economic literature to explain the concepts of scarcity and innovation in economic growth theory and modeling practices. The theoretical framework also serves to justify the model adaptations and parameterizations by drawing on ecological economic critiques against these theories. Section 4 then provides a background to the modeling frameworks by Golosov et al. (2014) and Chazel et al. (2023) and introduces the key adaptations to the model in line with Keen et al. (2019). The exact setup of the current model is outlined in Section 5 and choices on the algorithm and calibration in Section 6. The different model simulations and their results are then presented in Section 7, which shows the impact of reframing the role of energy and including mineral constraints. The sensitivity of these results to changes in three key parameters is tested in Section 8. Lastly, Section 9 will provide a discussion of the model limitations and recommendations for further research.

2 Minerals, scarcity, and the energy transition

Although the main aim of the thesis is to reconceptualize energy in production by changing the production function and integrating mineral constraints, there are some important technical concepts and definitions when talking about minerals and the energy transition. The following section will therefore provide a brief technical background, before moving on to describe more general economic theories in Section 3.

2.1 Critical minerals and the energy transition

An energy transition simply means a shift from one pattern of energy to a new system of energy, including energy sources, carriers, and services (Smil, 2017). There have historically been many energy transitions, and most of them were slow processes of technological innovation and diffusion spanning multiple decades or even centuries. In the context of climate change, the energy transition has been understood as the shift from fossil-based energy to "renewable" energy, with the aim of decarbonizing the energy supply. However, despite not requiring fossil fuel minerals such as oil and coal, low-carbon energy production is resource-intensive in other ways and heavily depends upon the use of raw materials. A large variety of minerals are required as inputs for the technologies necessary to convert the energy potential of sun, wind, and water into actual usable energy.[1] Of these minerals required for the energy transition, many face increasing challenges that limit their availability.

Although the definition of what classifies a mineral as critical is also contested, the International Energy Agency (IEA) characterizes critical minerals as those that are essential for a range of energy technologies in the broader economy, and that face a high risk of supply chain disruptions (IEA, 2021). These risks, if not addressed, could make the energy transition slower and more costly, and thus hamper progress necessary to tackle climate change. In the model described in Section 5, the mineral used for calibration is copper, but the issue of scarcity in the energy transition is much broader than that. Following their definition, the IEA analyzes five critical minerals in their annual Global Critical Mineral Outlook: Copper, lithium, nickel, cobalt, graphite, and the group of rare earths (a subgroup of four rare earth elements, neodymium, praseodymium, dysprosium, and terbium).

As Figure 1 illustrates, different low-carbon energy technologies depend on a varying mix of critical minerals. Copper is essential for nearly all low-carbon technologies. Cobalt, lithium, and nickel are used for electric vehicles and battery storage, rare earth elements are used in consumer electronics and wind turbines, and green hydrogen, necessary to decarbonize heavy industry and

[1] There thus are renewable energy sources, but there might be no such thing as renewable energy when considering the non-renewable resource requirements of the necessary technologies (Dunlap, 2021). This thesis will therefore use low-carbon energy when referring to what commonly might be understood as renewable energy

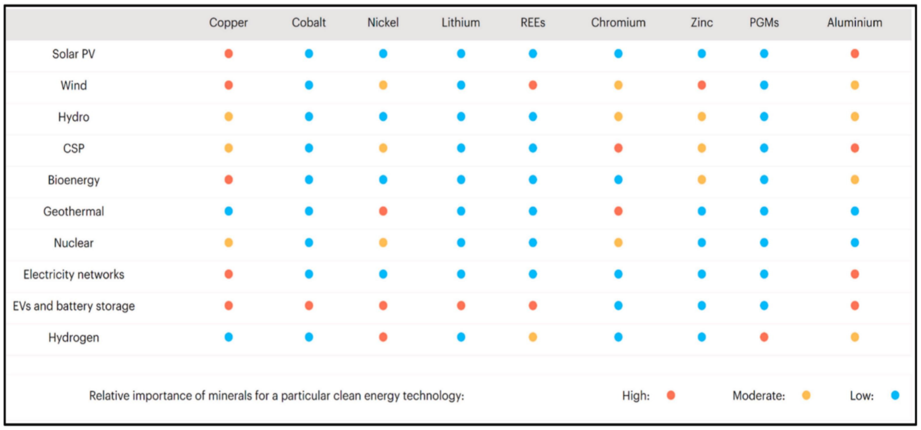


Figure 1: Critical minerals. From IEA (2021)

maritime transportation, depends heavily on nickel or platinum group metals IEA (2021). In turn, different minerals are characterized by different supply risks.

Supply-chain disruptions can range from physical depletion to vulnerabilities associated with environmental and geopolitical concerns. For instance, global mining areas for critical minerals overlap with protected wilderness areas to a larger extent than areas used for fossil-fuel related mining, and geopolitical tension and the ongoing trade war between China and the US have led China to introduce export controls on several key minerals (IEA, 2024a; Sonter et al., 2020). Overall, all these issues impact the energy transition through three channels: by increasing price volatility, reducing availability, and increasing uncertainty. This highlights the importance of including mineral prices and mineral stocks when modeling the energy transition.

2.2 Projections of mineral scarcity

Leaving geopolitical factors aside, the most imminent challenge for most critical minerals is not depleting resource reserves, but that demand exceeds the available supply. These represent two different concepts of scarcity that are important to distinguish: physical scarcity and economic scarcity.

In a physical sense, there exists a fixed and finite quantity of critical minerals on earth. This *base resource* denotes the mass of each resource that is estimated to exist in the earth's crust (Perman et al., 2011). However, the majority of this resource exists in very dispersed forms, or at locations that cannot be retrieved under current technologies. Therefore, whether or not a resource is physically scarce does not necessarily determine whether there will be a shortage of this resource within a foreseeable amount of time. What matters to economists is not only what is technically feasible, but also what is economically viable. Where the base resource is a purely physical measure, *potential reserves* quantify how much we can extract from the earth given certain assumptions on current and future technology (Perman et al., 2011).

A mineral like copper is relatively abundant from a purely physical perspective, ranking as the 26th most abundant element in the earth's crust. Yet in their 2025 Critical Mineral Outlook, the IEA still projects a gap between supply and demand in as little as 10 years (IEA, 2025). Under current technologies, the projected supply of copper from existing and announced projects is lower than the amount of copper required to meet climate targets by 2050. Taking a long-term perspective, if everyone in the world would consume as much as current Global North citizens, copper might be physically depleted in as little as 100 years (Henckens, 2021). Both these projections imply that increased development is needed to drive an expansion in exploitable reserves, or a major expansion in efficiency and recycling (ETC, 2023).

To target the latter, circular economy strategies aim to reduce the impacts of scarcity by minimizing primary resource extraction and waste, for instance, by increasing the recyclability of certain materials. Nevertheless, a fully circular supply chain for low-carbon energy may be difficult to achieve, as only about one third of metals have more than a 50% recyclability, meaning they cannot be recycled repeatedly without loss of performance. The recyclability of some key minerals is even below 1% (Graedel et al., 2011). Furthermore, despite its high recyclability, the share of secondary copper in the overall copper supply has not improved since 2015, mainly because increases in the recycling rate are offset by increases in demand (IEA, 2024b). This suggests some level of primary mineral extraction will remain necessary in the foreseeable future.

Important to note is that there is a wide range of projections on the future scarcity of minerals. For instance, Wang et al. (2022) find that critical mineral demand for wind and solar power is projected to increase between 2.6 and 267-fold. This is in part because leading organizations make projections based on a varying set of assumptions around mineral efficiency, recyclability, and the assumed life span of different technologies, but they often lack an inclusion of mineral prices, meaning they may not accurately reflect market mechanisms that can mitigate resource scarcity (Pauliuk et al., 2017; Wang et al., 2022).

Simply put, most projections from a technical perspective analyze the amount of minerals necessary to supply a specific set of clean energy technologies, without looking at price mechanisms or whether the currently available mineral supply would be able to meet demand (Hund et al., 2022). In economic theory and modeling, the price mechanism is key in projecting long-term pathways of non-renewable resource extraction. The prevailing idea, mainly within neoclassical frameworks, is that challenges around scarcity and a lack of available technologies can be resolved through innovation and substitution. The following section will discuss these theories and the critiques against them, to justify choices in the later model simulations.

3 Theoretical framework

This section will provide a theoretical background to the research question. Section 3.1 briefly touches on the history and concept of scarcity in economics, and how this relates to theories of nonrenewable resources and economic growth. Two important concepts for this relationship between resources and growth are total factor productivity and substitution. To briefly summarize, the prevalent idea in economics is that historically and currently, most economic growth can be attributed to innovation - or growth in total factor productivity - i.e. producing more output with the same inputs. Section 3.2 will outline these ideas by drawing on more standard growth theories, and touching on how their concepts influence current environmental-economic models (including the model framework used in Section 5). But these prevalent notions are being challenged by a growing field of research related to post-growth and ecological economics, outlined in Section 3.3 & 3.4. Attempting to bridge the gap between the laws of thermodynamics and the "laws of economics", the main debate of interest for this thesis is whether the biophysical basis of economic activity implies that there are limits to growth. The following section will highlight the existing theoretical contributions relating to this central question.

3.1 Scarcity and nonrenewable resources in economics

Although the field of environmental economics is considered to be relatively young, the relationship between economic activity and resource scarcity has long been the subject of debate. For most of human history, the size of our populations was mainly constrained by the food supply. Given this context, early economic work had a central role for agricultural productivity and land use. So when Malthus (1798) wrote *An Essay on the Principle of Population*, he developed a theory which stated that the natural tendency of a population to increase exponentially is constrained by diminishing returns in agricultural production. His theory is not like the environmental economics we know today, but it laid the groundwork for awareness on the feedback between the environment, human productivity, and welfare (Sandmo, 2014).

Nearly a century later, Jevons (1866) wrote a book about his concern that England's economic progress was tightly linked to growth in industries that depended on coal as their source of energy, and given that the rate of extraction would decline over time, the increasing scarcity of coal would act as a constraint on the country's economic growth. Around the same time, a variety of economists contributed to our current understanding of environmental and natural resource economics. These include but are not limited to the theory of utility maximization in general equilibrium (Walras, 1877), allocative efficiency (Pareto, 1909), and externalities (Pigou, 1920). An important leap forward came with the work of Hotelling (1931) who argued for the "rule" that under perfect competition, the net price of a nonrenewable resource must grow at a rate equal to the rate of interest. This also implies that the price of a resource will rise with its scarcity, as a declining stock will make the resource more valuable. Interestingly, his analysis came as a response to the rising conservation movement at the time, who felt that the price of resources was now so low that they were being exploited at a rate too fast to serve future generations (Sandmo, 2014).

As these concerns around declining natural resource stocks grew, perhaps the most famous work on biophysical constraints to economic activity came in 1972 with the book *The Limits to Growth*. In this work, a team of researchers at MIT developed a system dynamics model, where they identified five factors that impact the earth's system. These factors were presented as determining and ultimately limiting growth: nonrenewable resource depletion, industrial output, pollution generation, agricultural production, and population increase (Donella H. Meadows et al., 1972). The outcome of the Standard Run of their model was quite bleak. As industrial capital grows, it will require an increasing amount of resources, until the depletion of these resources will lead to the collapse of industry, followed by a collapse of the economic activities that depend on it, like the food supply, which will eventually lead to the collapse of the human population. It seemed to tell a Malthusian story, where when the capacity of the earth reaches its limit, the human population will suffer, and ultimately collapse.

But as Malthus was seen to be incorrect about human population being constrained by the trap of agricultural production, and Jevons was seen to be incorrect about the decline of coal leading to the demise of growth for Britain, so were the ideas of Donella H. Meadows et al. (1972) seen as being too pessimistic, and not taking into account key economic mechanisms. To conclude, despite centuries of concerns around resource scarcity and diminishing returns, the global economy was and is showing continuous growth. How could this be possible?

3.2 The role of innovation and substitution

There is a large variety of economic theories aiming to explain long-run economic growth. A look at the chapters of the *Handbook of Economic Growth* can give an idea of the wide range of explanatory factors, from culture and family values to institutions and entrepreneurship (Philippe Aghion & Steven Durlauf, 2014). Leaving valuable qualitative insights aside, a key development in our quantitative understanding of economic growth came in the late 1950s with the Solow-Swan revolution. Independent of each other, they created mathematical models that aimed to explain aggregate economic growth (Solow, 1957; Swan, 1956).

One of the most important ideas in their models is the adoption of a Cobb-Douglas *production function*, a mathematical expression that illustrates how the inputs of labour and capital are combined to produce economic output. The intuition behind this was simple but groundbreaking: increasing any input

- independent of the other - would increase output, but the more you add of one input, the smaller its impact on output becomes. Think of it in terms of an assembly line; increasing the number of workers from 10 to 15 per line might initially increase production by providing a better division of tasks, allowing you to produce goods more efficiently. But as more and more workers are added, and all necessary tasks are fulfilled, adding five additional workers will no longer increase output by much. This production function therefore basically reflects the Malthusian idea of diminishing returns (Susskind, 2024).

Without going into technical detail on the exact formulation of the production function (for this, see Section 4.3) the main intuition that followed from the Solow-Swan model is that in light of diminishing returns, economic growth can only be sustained through continuous technological progress. As demonstrated by the above example, adding more labourers or more machines will increase production at a diminishing rate. Therefore, in order to achieve sustained growth, it is not enough to simply add more of each input, it is also necessary that these inputs become more productive over time. If one worker produces 10 units of output per hour, training them to work more efficiently might lead them to produce 15 units per hour. The same could be said about investing in newer, more efficient machinery or automating tasks using AI. In growth models, these innovations are often captured by an exogenous parameter called total factor productivity (TFP) which explains the changes in output that are not directly related to changes in inputs.[2]

A second important way in which innovation can lead to growth is through its impact on substitution. Where total factor productivity relates more to the overall efficiency of inputs in producing output, technological progress can also enable a switch from one input to another. Substitution can be between the inputs of capital, labour, and energy, or it can refer to substitution within an input, such as switching from coal to gas. The mechanism underlying this form of substitution posits that the depletion of a non-renewable resource will over time lead to higher prices, in line with Hotelling's rule, which in turn incentivizes producers to increase efficiency or to switch to alternative inputs (Perman et al., 2011). This makes the production process more resilient. Thus, innovation is a tool to make inputs into production be used more efficiently and productively, which can offset the negative economic impacts of scarcity and diminishing returns, allowing for a continuous positive rate of economic growth, also characterized as a *balanced growth path*(Susskind, 2024). For this reason, technological progress is often characterized as the most important driver of long-run economic growth (Smulders & Withagen, 2012).

In the case of the energy transition, the concepts of innovation and substi-

[2]Later theories aimed to endogenize technological progress by accounting for factors such as human capital or knowledge and ideas (Susskind, 2024). Although important in our understanding of economic growth, they do not stand in contrast to the earlier notion that innovation and substitution (whether in human or physical capital) are the main driver of growth, and thus they will not be separately discussed.

tution are especially relevant. As prices for fossil energy rise, companies are incentivized to innovate or substitute. For instance, they can invest in more energy-efficient materials, or switch to solar electricity. Yet it is important to note that at the time of the earlier Solow-Swan model, nonrenewable resources were not included as a factor of production. This inclusion came some years later, in the time of - and perhaps as a critique against - the report by Donella H. Meadows et al. (1972). Amidst heightened concerns about the volatility of the oil supply, neoclassical economists set out to better account for nonrenewable resource use in their growth models, and nonrenewable resources were added as a separate input in the production process, alongside labour, capital, and TFP. Important contributions to this came from Solow himself (Solow, 1974), as well as Dasgupta and Heal (1974) and Stiglitz (1974). Although each of these works contributed in its own way, their shared understanding remained that innovation and substitution can prevent the societal collapse as predicted by *The Limits to Growth*.

3.3 Energy, exergy, and thermodynamics

In opposition to the neoclassical work from Solow and others, there have been historical and contemporary critiques against the underpinnings of their models and theories, specifically when applied to environmental problems. The main arguments of interest came from ecological economists who argue that there are limits to substitution and limits to technological progress, in line with the laws of thermodynamics. This inclusion of physical laws to conceptualize economic processes can be traced back to early foundational works by Daly (1968), Ayres and Kneese (1969) and Georgescu-Roegen (1971). Often stemming from a background in physics and engineering, their main aim was to model the economy in biophysical terms, based on flows of energy and matter, where the conversion of materials into final goods inherently requires energy and leads to waste and pollution (Røpke, 2004). In essence, their critique boils down to the question of whether the economy is constrained by the earth's physical limits, or if these limits can be changed or overcome by human ingenuity. To better understand the basis of their arguments, it is necessary to briefly delve into the concepts of energy, exergy, and the laws of thermodynamics.

The concept of energy is hard to define (a loose definition would be the capacity to do work) but the laws of thermodynamics that govern how energy behaves are very explicit. The First Law of thermodynamics, also known as the law of conservation of energy, states that energy cannot be created or destroyed, meaning that within a closed system, the total amount of energy is constant over time. The Second Law of thermodynamics, also known as the law of entropy, states that every energy transfer that takes place in a closed system will increase entropy (or disorder) of that system, and reduce the amount of energy available to do work. In other words, no conversion of energy can be 100% efficient. This may all seem rather complex, but it becomes quite intuitive by taking the example of the steam engine.

The coal-powered steam engine fueled the first wave of industrialization. By burning coal in a firebox, water is heated, creating high-pressure steam, which is used to move pistons in a cylinder, creating motion. This transforms the chemical energy contained in coal into mechanical energy. Although groundbreaking, the conversion of energy in this process was highly inefficient. Coal has an average stored energy content of let's say, 30MJ per kilogram, but when this is burned it does not provide an equivalent of 30MJ in available energy. The usable amount of energy is also called *exergy*, which represents the maximum amount of energy that is available to do useful work, such as moving the wheels of a locomotive.[3] When fueling the steam engine, more than 90% of the energy stored in coal is wasted as heat and pollution. Some energy is lost when the boiler heats the water, and around 65% is lost through steam escaping the cylinder.

In line with the First Law, the total amount of energy in the system does not change. After burning 30MJ in the form of coal, 3MJ are available as exergy, and the other 27MJ are dissipated into the environment as low grade heat (such as steam and fumes). In line with the Second Law, although the quantity of energy remained the same, the quality of that energy did not. Energy lost as low-grade heat is no longer usable. This also shows that to some extent, the amount of stored energy that can be transformed into useful work depends on the available technologies. For instance, later steam engines were much more efficient, as the boiler did not let as much steam escape. Yet the conversion of energy to exergy can never reach 100%, as the Second Law dictates that every energy transfer that takes place will increase entropy, meaning some energy is wasted.

Bringing this back to the economy, critics of standard economic theory highlight that these same principles apply to all economic activity, making energy a prerequisite for economic processes. Stern (2011) summarizes this by arguing that, as all work requires energy, and all production involves work, all production must require energy, and there must be limits to the substitution of other factors of production for energy so that energy is always an essential factor of production. I will return to this argument later, when justifying changing the Cobb-Douglas production function as described in Section 4.4. In sum, whether it is a machine producing microchips, a programmer training an AI model, or a husband picking up the kids from school, all work requires energy and is thus governed by the laws of thermodynamics.

3.4 Energy and historical economic growth

As energy is seen as a prerequisite for economic activity from a biophysical perspective, ecological economists reframe economic growth as a process that fundamentally depends on energy (Ayres & Warr, 2009, Chapter 1). In this

[3]From here on forward, I will use useful energy, usable energy, useful work, and exergy interchangeably.

energy-centered perspective, technological progress is not an independent source of growth, but rather an instrument that allows us to channel energy more effectively. Economic growth is then not only constrained by the availability of labour and capital, but primarily by the available quantity and quality of useful energy that can be captured to produce output. This offers an alternative perspective on historical economic growth.

For most of human history, growth, in economic and demographic terms, was inherently constrained by the food supply. Societies depended largely on the energy of the sun to grow food and fuel wood, and on animate sources of energy from humans and animals to work the land Smil (2017). Growth was possible in times of *energy surpluses*, where useful energy remained after accounting for the energy costs of agricultural production (Hall & Klitgaard, 2012). In this regime, production was limited by the availability of land and labour to transform energy into agricultural output, and marginal returns to labour were low. This was the world Malthus experienced when he developed his theory. Sources of energy were abundant, but the capability to harness it and turn it into useful work was not.[4]

This all changed with the onset of the Industrial Revolution. From the 1750s onward, a series of technological innovations changed the way we use energy, allowing the human population to escape the Malthusian trap of diminishing returns. The shift from wood to coal as the main fuel for production created a new regime, where energy was cheaply and abundantly available, and could be transformed into useful work much more efficiently. The limiting factors were no longer land and labour, but rather capital's ability to harness energy to perform useful work. The marginal benefits of increasing capital's productivity were high - think of the use of the Spinning Jenny, converting the energy exerted by the human body into cloth much more efficiently than a spinning wheel - and countries that extracted high rents from successfully expanding the energy productivity of capital achieved rapid growth (Allen et al., 2019).[5]

During this rise of industrial output, Solow and Swan saw large increases in output that were not explained by the accumulation of labour and capital alone (Solow, 1957). This unexplained part of growth was attributed to technological progress and later captured by the term TFP. As growth continued with the discovery of oil and the introduction of the internal combustion engine, this idea of TFP as a driver of growth was further supported. However, following the oil crisis, the model was critiqued for not including nonrenewable resources as a factor of production, energy was added as another substitutable input, along-

[4]There were of course important developments in the way energy was channeled to produce output before the Industrial Revolution, for instance through the use of waterwheels and windmills, but the energy improvements these innovations presented were incremental, and not enough to create the consistent and increasing surpluses necessary to support long-run economic growth (Smil, 2017).

[5]Characterizing inventions like the Spinning Jenny as enhancing the productivity of capital rather than the productivity of labour is an important conceptual departure from standard economic thinking, and an in-depth discussion of this is provided in Section 4.4.

side labour and capital (Stiglitz, 1974). Even at the time, this approach was criticized by ecological economists such as Georgescu-Roegen (1979), who argued that simply adding energy to the equation failed to recognize "the physical finitude and the irrevocable exhaustibility of natural resources."

From an ecological economic perspective, all major accelerations of economic growth coincided with changes in the quality and quantity of energy available to production (Smil, 2017). A growing share of empirical research supports this strong link between energy and economic growth (Flament, 2023; Giraud & Kahraman, 2014). Others specifically address it is not energy as such, but how energy sources are transformed into usable energy (exergy) that drives economic growth (Ayres & Warr, 2005; Ecclesia & Domingos, 2024; Gasparatos et al., 2009).[6] The steam engine, for instance, allowed a given amount of coal to be transformed into usable energy much more efficiently than burning wood. The internal combustion engine again enabled higher qualities and quantities of usable energy, as oil is highly mobile and more energy-dense than coal. This powered cars, trucks, airplanes, and machinery, which were all essential in supporting rapid economic growth.

Growth thus depends on the availability and productivity of energy rather than on exogenous increases in TFP. Innovations were crucial in achieving past economic growth, but the technologies themselves enabled growth insofar as they expanded the energy supply and/or enhanced the productivity of energy-using devices (Hall & Klitgaard, 2012; Palmer & Floyd, 2025). This reinterpretation of past economic progress also has implications for the future of sustainable economic growth.

3.5 Technology and the future of economic growth

With the depletion of resource reserves and growing awareness around the environmental impact of fossil fuels, we have arguably entered a new regime where energy is no longer the cheap and abundant input, even though it is often modeled as such. In this new regime, where the energy supply is becoming the limiting factor, countries that benefit are those who can capitalize on the energy supply or those who can increase the value of their output per given amount of energy. Nevertheless, if energy is a prerequisite to economic production, constraints to energy would mean that there are inevitable limits to growth. This question is at the core of the *decoupling* debate, which centers around whether the economy can continue growing, while energy use and its associated environmental impacts decline. Technological innovation again plays a key role: Where past innovation allowed capital and labour to harness larger quantities of energy, future innovation should reduce the use of energy while maintaining positive long-run economic growth.

[6] An important note is that although the strong relationship between energy use and economic growth is quite well established in the literature, there is currently no consensus on the direction of causality nor on the underlying factors (Palmer & Floyd, 2025).

The prevailing idea among economists remains that as human ideas are endless, so is economic growth (Susskind, 2024). The strength of this idea is exemplified by this year's Nobel prize in economics being awarded to Joel Mokyr for identifying knowledge and ideas - rather than resources and incentives - as the main driver of sustained growth through technological progress (Mokyr, 2005). In the context of the service economy and the rise of artificial intelligence, scholars like Mokyr argue that the economy has outgrown its physical basis. Bottleneck issues around scarcity in the energy supply never arrived, and the main concern around energy use is now its contribution to CO_2 emissions. From this "techno-optimist" perspective, these issues can once again be resolved through technology. Increasing efficiency and circularity can reduce the energy intensity of production, and price mechanisms such as carbon taxation can steer investments towards cleaner technologies.

Acemoglu (2002) describes this mechanism by distinguishing between capital-directed and energy-directed technical change. When energy is cheap, innovation typically targets capital's ability to use energy, but this comes with an *energy bias*, meaning it increases the energy throughput per unit of capital. Where energy is scarce (or more costly due to its impact on the environment) innovation is typically targeted at energy saving technologies, reducing the energy intensity per unit of output. The success of decoupling depends on these energy-saving technologies, such as green hydrogen electrolysis or nuclear fission, as they allow us to create economic value while reducing the impacts of increased energy usage.

While these innovations are possible in theory, improvements in energy efficiency have not led to an absolute reduction of energy and material requirements at a global scale. This pattern might be explained by Jevons' paradox, which posits that the decline in price of a resource associated with efficiency increases leads to increases in overall consumption (Jevons, 1866). Furthermore, many technologies necessary to achieve climate ambitions do not yet exist, or are not yet deployable at scale. Even if they do become available, they are expected to have huge mineral requirements. In Europe alone for instance, the iridium demand for green hydrogen electrolysers is expected to require 87% of the current worldwide supply by 2035 and 122% by 2050 (Elzenga et al., 2025; Wieclawska & Gavrilova, 2021). Even the digital economy has a large material footprint. If data centers represented a country, it would be the 11th largest electricity consumer in the world, between the nations of Saudi Arabia and France (Bashir et al., 2024). Ecological economists therefore remain skeptical of the promises of technological innovation to reduce overall energy and material use.

If energy is indeed a prerequisite for economic growth, this has some important implications for the future. First, it implies that economic production is limited by the quality and quantity of available energy (Giraud & Kahraman, 2014; Stern, 2011). The growing scarcity of fossil fuels and minerals then constrain economic growth through their impact on the energy supply. Yet it is not the availability of energy alone that determines economic growth, but also the productivity of capital to transform this energy into output. This complementary relationship between energy and capital stands in stark opposition to the neoclassical idea that energy can largely be substituted for by labour and capital. Second, when energy and capital are complements, economic growth is constrained by whichever is scarcest. This implies that growth can occur either through increasing the capacity of capital to process energy (which is energy-biased) or by increasing the usable energy supply (which is energy-saving). There are thus two opposing forces at work, one that increases overall energy use and one that reduces it. Whether sustainable long-run economic growth is possible then greatly depends on the speed and direction of future technological innovation, and whether the energy gains from energy-saving technologies outpace the growth of energy-increasing capital throughput.

The energy transition thus presents an interesting case for analysis, as it connects these age-old questions around scarcity, energy, technology, and economic growth. It links Solow's optimism on technological innovation, Hotelling's theories about resource depletion, and Georgescu-Roegen's work on thermodynamic constraints. This thesis then formalizes these concepts by (i) using a production function in which energy and capital are complements and by (ii) explicitly integrating mineral constraints in the production of low-carbon energy. By making various assumptions on technological progress, the following model examines if and how innovation can offset the impacts of scarcity and diminishing returns, while simultaneously increasing growth and reducing emissions.

4 Background to the current model

The current model is inspired by the work of Chazel et al. (2023), which builds on the model by Golosov et al. (2014). Golosov's GHKT model can again be seen as an adaptation and extension of Nordhaus' famous DICE model. This begs a brief description of the models and their differences.

4.1 DICE (1992), GHKT (2014), and Chazel (2023)

Where in the early 1970s the inclusion of fossil energy resources in economic models served mainly to understand the impact of its depletion, this was later extended to include the relationship between fossil fuels and their impact on climate change (Smulders & Withagen, 2012). The inclusion of energy as a factor of production in macroeconomic models gave way for its application to environmental-economic analyses, including many integrated assessment models. One of the earliest of these models is the DICE model. First published by Nobel laureate William Nordhaus in 1992, DICE is one of the most widely used IAMs to assess the social cost of carbon (SCC) and to evaluate climate policies (Greaves, 2020). It adopts a common neoclassical growth model integrated with a damage function, an abatement function, and equations to relate GDP growth to CO_2 levels and the impact of those CO_2 levels on average global temperature. These climate-related functions link economic activity to global warming, which in turn negatively impacts welfare. It can be characterized as a growth model, but it aims to provide a macroeconomic overview of the interplay between the economy and the environment. DICE does, however, share many of the same theoretical underpinnings as the Solow-Swan model, including a Cobb-Douglas production function, in which innovation and substitution remain paramount to achieve sustainable economic growth.

Golosov et al. (2014) state that the spirit of their modeling is entirely in line with the approach used by Nordhaus, but they depart from DICE in several important ways. The most relevant departure from the DICE model, in line with their aim to identify the optimal carbon tax, is that their dynamic stochastic general equilibrium (DSGE) model, called GHKT, explicitly includes three different sources of energy: coal, oil, and low-carbon energy (taken to be wind and solar energy). Integrating different energy sources and their respective climate impacts allows for the modeling of optimal extraction paths over time. A second key difference is that GHKT makes different assumptions about the depreciation structure of carbon. They assume what matters for global temperature increases is not the carbon concentration over time but rather total cumulative emissions, and that impacts of CO_2 emissions on temperature are immediate (as opposed to Nordhaus, who assumed that the oceans create a drag on temperature increases). Their carbon depreciation structure leads to significantly larger effects of emissions on the climate compared to the DICE model.

Important to note is that the implication of their model specification is that it produces a rather simple function for the social cost of carbon in which the damage - and thus the optimal carbon tax - is a fixed proportion of GDP. This proportion depends on only three basic parameters: the discount rate, the expected damage elasticity, and the structure of carbon depreciation in the atmosphere.[7] This means that other values such as future output, consumption, population, technology, productivity, and energy sources all disappear from the formula. Under these assumptions, they find that the optimal carbon tax is higher than in the DICE model and that coal should be phased out sooner due to its high impact on the environment.

Given the disaggregated energy production sector, Chazel et al. (2023) take the GHKT model as a starting point for analyzing the impacts of mineral constraints on the energy transition. They treat the baseline GHKT model as an "infinite mineral scenario", and then change this by conceptualizing green energy as a non-renewable energy source due to its dependency on non-renewable mineral inputs. They aim to investigate the implications of primary mineral constraints and recycling on the optimal path of low-carbon energy production (and fossil-fuel extraction) and how this in turn impacts GDP compared to the unconstrained GHKT benchmark. Applying these constraints and calibrating the model with data for copper, they find that mineral constraints limit the development of green energy. In terms of the timing of the transition, they find that green energy will reach a peak in production before eventually hitting a

[7]See Golosov et al. (2014) Section 2.3 *The Planning Problem*, equation 11.

plateau, the exact time of which depends on assumptions around the depreciation of green capital and the recyclability of primary minerals. Overall, Chazel et al. (2023) conclude that GDP will be lower compared to the benchmark case, but that it continues to grow due to increases in labour productivity.

4.2 The rationale behind the models

Despite their differences, the rationale behind the models is the same. At the microeconomic level, households derive utility from consumption, and they can choose to consume now or to save money for later. Saving is an investment into the future, but the future is uncertain, so there is a tradeoff between enjoying what you have now or investing with the expectation of enjoying greater consumption in the future. At the macroeconomic level, the same logic applies to the economy as a whole. The economy produces output based on capital, labour, and energy. A part of this output is consumed, and a part of it is invested and creates productive capacity for the future. If people save too much, it means there is little utility from present consumption, but saving too little means there is less capital to produce output to be consumed later. A social planner who wishes to maximize welfare over a longer period of time will thus choose an optimal savings rate that finds the right balance between consumption and investment. This picture is somewhat complicated with the introduction of climate change, but the overall idea stays the same.

Economic growth arises from this process. When part of today's output is invested, the stock of capital expands, enabling higher production to be consumed in the future. However, as explained before, sustaining this growth depends on how energy, capital, and labour are combined to produce output. This relationship is captured in a production function. The main similarity of interest between all three models is that they adopt a Cobb-Douglas production function extended to include nonrenewable resources as an input (based on Cobb and Douglas (1928), extended by Solow (1974) and Stiglitz (1974)). Changing the Cobb-Douglas specification is the main contribution of this thesis but before doing so in Section 5, it is necessary to first touch on its limitations.

4.3 Cobb-Douglas and its limitations

The standard specification of Cobb-Douglas, omitting the damage function but including energy, is as follows:

$$Y = AK^{\alpha}L^{1-\alpha-\upsilon}E^{\upsilon}$$

Here, Y is output, and the inputs are capital K, labour L, and energy E. The exponents α, υ, and $1-\alpha-\upsilon$ represent the output elasticities of each input, and the level of technology is indicated by A, also referred to as total factor productivity (TFP). Using this production function is common practice in environmental economic models, as Cobb-Douglas has certain properties that allow for tractability and are useful in linking microeconomic behaviour to macroeconomic outcomes (Michaelides, 2024).

First, when the sum of its exponents equals one, Cobb-Douglas is homogeneous of degree one, meaning that the outcome of the function will exhibit constant returns to scale, whereby increasing all inputs by a certain factor should lead to an increase in output by the same factor. Secondly, the input coefficients in the Cobb-Douglas production function represent their output elasticities. An output elasticity of 0.04 for energy, as adopted by Golosov et al. (2014) and Chazel et al. (2023), means that a 1% increase in energy inputs is associated with a 0.04% increase in output, all else equal. In practice, these elasticities are often derived applying the "cost-share theorem", which dictates that under profit maximization and perfect competition, the output elasticities equal the input's share in the total cost of production.

This specification of the Cobb-Douglas production function is not free from scrutiny, and even Golosov et al. (2014) state themselves that Cobb-Douglas is perhaps one of their weakest assumptions. The arguments pointing to its limitations are varied, but for this thesis the above properties are what matter. Although it is clear that a standard Cobb-Douglas production function (without the addition of energy as an input) has desirable properties from a mathematical and economic viewpoint, some issues emerge when the standard function is extended to include energy as an input. Keen et al. (2019) lay out key concerns with the Cobb-Douglas production function. Their conviction is that energy plays a non-essential role in this equation, and that the implications of this fail Solow's smell test.[8]

To begin, many properties of the Cobb-Douglas production function depend upon the assumption of unitary elasticity of substitution. This implies that inputs are moderately substitutable and that a relative change in prices will lead to the same relative change in the inputs, all else equal. This assumptions seems reasonable when looking at the inputs of capital and labour. For instance, if the price of capital increases by 10% relative to the price of labour, the input of capital should decrease by 10% relative to that of labour. However, with the addition of energy as a separate factor of production, some issues emerge. If energy prices increase by 30% relative to capital, the input of capital should increase by 30% relative to that of energy. Evidence for this mechanism is limited. Some case studies drawing on the 2022 European energy crisis to test this relationship, find that facing an increase of relative energy prices, instead of substituting capital for energy, firms substituted away from natural gas towards other energy sources (Alpino et al., 2025). Beyond the substitution of energy, meta-analyses on the elasticity of substitution between labour and capital alone

[8] In a Congressional hearing, *Building a Science of Economics for the Real World* (2010) Solow stated that "When it comes to things as important as macroeconomics, I think that every proposition has to pass a smell test: Does it really make sense? And I don't think that the currently popular DSGE models—I can say Dynamic Stochastic General Equilibrium without a lapse—I don't think that those models pass the smell test." See also Solow and Touffut, 2012.

are estimated to be below 1, ranging from 0.5 to 0.9 (Gechert et al., 2022; Knoblach & Stöckl, 2019). This seems to support the idea that at least in the short term, the assumption of unitary substitution between energy and other inputs lacks empirical basis.

The second issue relates to the input coefficients in the Cobb-Douglas production function representing their output elasticities. As explained before, an output elasticity of 0.04 for energy means that a 1% change in energy inputs is associated with a 0.04% change in output, all else constant. These elasticities are often calibrated based on the cost share theorem, dictating that under perfect competition, the output elasticity of energy equals the inputs share to production. In calibration, this is often taken as the expenditure on energy as a share of GDP. This share is relatively small, around 4% to 7% in most economies, because the price of energy has historically been low compared to other inputs (perhaps due to its relative abundance or fossil subsidies or the difference between the marginal private cost and the marginal social cost of energy production). A low output elasticity means that energy only plays a trivial role in production (Keen et al., 2019). For example, an output elasticity for energy of 0.04 implies that a 50% increase in energy input would only increase output by 1.6%.[9] Subsequently, a fall in energy inputs of 10%, as was witnessed in the US during the 1973-74 oil crisis, should only translate into a 0.4% reduction in output. In reality, however, US gross national product fell by 2% and global output by 3% . These implications of the current Cobb-Douglas specification stand in stark contrast to the notion that energy and economic growth are tightly linked. Thus, while energy's contribution to GDP has historically been small in monetary value, its importance from a biophysical perspective implies that output is far more sensitive to fluctuations in energy availability than the Cobb-Douglas function suggests.

The last main element of controversy is the parameter A for Total Factor Productivity, also called the "Solow residual" after Solow (1957). As highlighted in the theoretical framework, in the production function TFP is an exogenous parameter that captures the changes in output that are not directly related to changes in the inputs of capital, labour, or energy. It is a measure of overall productivity, reflecting the share of output growth that cannot be explained by the contribution of inputs. It therefore is assumed to capture factors such as innovation, technological progress, social capital, and the quality of institutions (Argentiero et al., 2021). In standard growth theory, TFP is seen as the main driver for sustained long-run economic growth, but in essence, TFP is a kind of black box - an unexplained residual - as the variables seen to impact TFP are often hard to measure or predict. In the baseline models of Golosov et al. (2014) and Chazel et al. (2023), initial TFP is calibrated using data on global income, energy consumption, and an estimate for the capital stock, to determine what level of output is not explained by its inputs. This determines the initial level

[9] If the output elasticity of energy is $\gamma = 0.04$, then increasing energy inputs by 50% ($E_1/E_0 = 1.5$) raises output by only $(1.5)^{0.04} - 1 \approx 0.016 = 1.6\%$.

of TFP, which is then also usually allowed to increase at a certain rate per year, meaning it exogenously drives economic growth.

The concept and use of TFP in growth models has long been critiqued, for instance by those who aim to endogenize innovation (Acemoglu et al., 2012; Aghion & Howitt, 1992). As explained before, a growing share of the literature is trying to reframe TFP from an unexplained residual to an explained variation in useful energy (Ayres & Warr, 2005; Ecclesia & Domingos, 2024; Gasparatos et al., 2009). If the role of energy is consistently underestimated due to the specification and calibration of the production function, a large portion of growth explained by TFP is potentially driven by increases in the quality or quantity of energy available to the economy. This warrants a reconfiguration of the production function.

4.4 An energy-based Cobb-Douglas production function

Keen et al. (2019) take the link between output and energy as a starting point for their critiques against the Cobb-Douglas production function. Given the arguments that the standard specification minimizes the role of energy in production, they propose a new formulation. This new production function conceptualizes capital and labour inputs as conditional on the input of energy. In other words, without energy there is no production, or, as Keen himself puts it, "Capital without energy is a statue, and labour without energy is a corpse."

The exact specification of this production function will change, but some important intuitions are derived from it, which is why it will be discussed here. Their reformulation addresses the concerns around the concept of energy in economic modeling, by making the role of energy nontrivial. In their Energy-Based Cobb-Douglas Production Function, they denote their final specification of the production function as:

$$Q_t = (K_t E^K_{x,t})^\alpha (L_t E^L_x)^\beta$$

In this production function, the inputs are in terms of *energy* and output, Q_t, is *exergy*. Leaving issues around the definitions of capital and labour aside, for modeling purposes Keen et al. (2019) distinguish between (1) the stock of capital and the amount of labour, (2) the average energy throughput of capital and labour, and (3) the efficiency with which they transform energy into useful work. All three elements are present in their production function.

Specifically, the inputs L_t and K_t represent the amount of labour and capital at time t. Although the amount of labour can change over time, the exergy exerted by the average labourer does not. As Keen et al. (2019) note that where energy consumption per head has risen dramatically, the amount of energy needed per worker to perform a task has changed little, as the human capacity to perform useful work has remained constant. If anything, innovations often reduce the somatic exergy necessary to perform a task: Writing a thesis with pen and paper and having to consult books in the library would undoubtedly require students to exert more exergy than when sitting behind a computer all day.

$E^K_{x,t}$ and E^L_x represent the actual exergy performed by capital and labour. They comprise the product of the average energy throughput of capital and labour, respectively, $E_{K,t}$ and $E_{L,t}$, and the efficiency with which these turn energy into usable energy, $e^L_{x,t}$ and $e^K_{x,t}$. Meaning:

$$E^K_{x,t} = e^K_{x,t} E_{K,t}$$
$$E^L_x = e^L_{x,t} E^L_x$$

Contrary to labour, the actual exergy performed by capital does vary with time. The efficiency by which machines convert energy into useful work, $e^K_{x,t}$, has perhaps increased exponentially, mainly by limiting energy losses (Keen et al., 2019). The average energy throughput of capital, $E_{K,t}$, is also time dependent, and it is obvious that the average machine today consumes an amount of energy several orders of magnitude larger than it did 50 or 100 years ago: Writing a thesis on a laptop consumes much more energy than using pen and paper or a typewriter.

The model below takes the key idea that energy is prerequisite to the production process, and that the actual exergy by capital has risen dramatically while that of labour has remained constant, and integrates them into the framework used by Golosov et al. (2014).

5 The model

In the current model, the GHKT modeling framework by Golosov et al. (2014) will serve as a benchmark to be further adapted by using an energy-based Cobb-Douglas production function, similar to Keen et al. (2019), and mineral constraints, largely in line with Chazel et al. (2023). This begins with an overview of the benchmark GHKT model and the changed introduced by Chazel. Section 5.2 describes the new production function, and Section 5.3 will take the Chazel specification as an inspiration for the introduction of mineral constraints.

5.1 The benchmark: GHKT (2014) and Chazel (2023)

The model simulation lasts T periods, where each period represents a time span of 10 years. The aim of the model is to maximize the total sum of discounted utility, with discount factor β= 0.985. The utility of a representative household's consumption, C_t, is specified according to logarithmic preferences as:

$$U(C_t) = ln(C_t) \tag{1}$$

Y_t is the total output of the economy and capital at time t is K_t. In every period, the production of final goods is shared between immediate consumption

C_t and savings K_{t+1} (meaning capital for the next period). Since the benchmark assumes full capital depreciation over one period:

$$K_{t+1} + C_t = Y_t \tag{2}$$

$$0 \leq C_t \leq Y_t \tag{3}$$

The benchmark production function is a standard Cobb-Douglas function. Final goods are produced using capital K_t, labour $L_{0,t}$ (where 0 indicates the final goods sector) and energy E_t. Total Factor Productivity is $A_{0,t}$, and α and υ are output elasticities of capital and energy:

$$Y_t = A_{0,t} e^{-\gamma(S_t - \overline{S})} K_t^{\alpha} L_{0,t}^{1-\alpha-\upsilon} E_t^{\upsilon} \tag{4}$$

This function is similar to the specification discussed before, with the addition of the damage function, $e^{-\gamma(S_t - \overline{S})}$. This damage function works as follows: The production of oil and coal contribute to output by producing energy, but their production also causes emissions and climate damages, which reduces final output. How much final output is reduced exactly, depends on the amount of emissions in the atmosphere relative to pre-industrial levels $(S_t - \bar{S})$ paired with a damage factor, γ. Carbon emissions accumulate in the atmosphere each period, after which some of them depreciate. The amount of cumulative carbon emissions, S, in the atmosphere at time at time t is:

$$S_t = \bar{S} + \sum_{s=0}^{t} (1 - d_s)(E_{1,t} + E_{2,t-s}) \tag{5}$$

Here, $1 - d_s$ is the amount of carbon that is left in the atmosphere s periods into the future. In the model, this remaining amount of carbon depends on three parameters namely (i) a share of carbon emitted into the atmosphere that stays in it forever, (ii) a share of emissions that exits the atmosphere immediately into the biosphere and the surface oceans, and (iii) a remaining share that slowly depreciates.[10]

Energy comes from three sources: oil, $E_{1,t}$, coal, $E_{2,t}$, and low-carbon energy $E_{3,t}$. Energy is produced according to a CES production function where inter-fuel substitution is represented by ρ , and κ_1, κ_2, and κ_3 indicate the relative efficiency of each energy source in the production of energy.

$$E_t = (\kappa_1 E^{\rho}{}_{1,t} + \kappa_2 E^{\rho}{}_{2,t} + \kappa_3 E^{\rho}{}_{3,t})^{\frac{1}{\rho}} \tag{6}$$

[10] For a full account of the model's carbon cycle see Golosov et al. (2014) section 2.2.3 *The Carbon Cycle.*

For the production of oil, the assumed labour costs are negligible compared to its scarcity cost and thus, in line with Hotelling's rule, the price and production of oil depend only on the available oil stock, $R_{1,t}$. The amount of oil extracted can never exceed the available oil stock, and the oil stock can never go negative. This gives:

$$E_{1,t} = R_{1,t} - R_{1,t+1} \tag{7}$$

$$0 \leq E_{1,t} \leq R_{1,t} \tag{8}$$

Unlike oil, coal is taken to be so abundantly available that its extraction costs dominate its scarcity cost. Assuming that coal extraction is solely constrained by labour, energy production from coal depends on the labour supply, $L_{2,t}$, and on labour productivity in the coal extraction sector, $A_{2,t}$:

$$E_{2,t} = A_{2,t}L_{2,t} \tag{9}$$

Where Chazel et al. (2023) make their distinctive contribution is in their assumptions around the constraints to low-carbon energy production. Golosov et al. (2014) state that clean or green energy sources are not associated with climate externalities. The GHKT benchmark framework modeled low-carbon energy production to only require labour, but the model by Chazel et al. (2023) expands this by explicitly modeling a primary and secondary mineral sector. An overview of how Chazel et al. (2023) models low-carbon energy production can be found in the Appendix under Section A.

Although these ideas serve as an inspiration, the current model uses a different pathway for the use of minerals in low-carbon energy production. To summarize the differences, my model assumes that mineral extraction costs are dominated by its scarcity rather than its labour costs, that primary minerals and labour contribute to green capital production (rather than minerals alone), and that green capital production *always* requires some - albeit small - level of primary minerals.

5.2 The new production function

Taking Golosov et al. (2014) as a starting point, the current model makes its novel contribution by integrating an energy-based Cobb-Douglas production function into the benchmark GHKT framework, inspired by Keen et al. (2019). As energy is no longer treated as a separate input in the production function, but instead as a prerequisite in the production process, I model output function Y_t according to a nested Cobb-Douglas production function, where the production of capital is conditional to the production of energy:

$$Y_t = min(E_{K,t}K_t, e_{x,t}E_t)^\alpha L_{0,t}^{1-\alpha} \tag{10}$$

In line with Keen et al. (2019), output is expressed in terawatt hours of usable energy. Doing so better reflects the laws of thermodynamics by taking into account that the stored energy that goes into production (such as tonnes of coal) does not equal the energy available to production, as some energy is always wasted.

The exact specification of Equation 10 deviates from Keen et al. (2019) but the intuition behind it is the same. There are two notable differences. First is the omission of energy as a prerequisite for labour. Although I agree that labour without energy is a corpse, the actual energy consumption and exergy contribution of labour to production are negligible compared to that of capital, and it is thus omitted for simplicity.[11] Doing so also supports the idea that it is the relationship between energy and capital that matters for modern-day economic production, not the relationship between energy and labour. Second, by explicitly modeling the relationship between capital and energy as one of perfect complements, in line with Leontief (1941), it ensures that a minimum level of energy is required for capital to be productive. It prevents the issue that energy can be a perfect substitute to capital, or that large reductions in energy have a minimal impact on output. In this model, a machine without energy would indeed be a sculpture.

Following the notation by Keen et al. (2019), $E_{K,t}$ refers to the average usable energy throughput of capital and K_t is the stock of capital at time t. Their product is the amount of usable energy that can be harnessed by capital at a certain time, expressed in terms of exergy. This is limited by the amount of exergy that is available to do work, which is the product of the flow of energy, E_t and the efficiency by which this energy can be transformed into exergy, $e_{x,t}$.

The limiting factor to economic output is thus either the total amount of energy available to do work, or the ability of capital to harness that energy. Using the same amount of capital but with a higher average energy throughput would require more energy to be available in order to increase output, which can either be obtained by improving the energy-to-exergy efficiency, which increases the amount of energy available to do work, or by expanding the overall energy supply. If the energy supply is fixed, adding more capital to the capital stock does not increase output without technological innovation, as it would simply reduce the energy throughput of the average machine. To increase output, it is

[11] Smil (2017) mentions in his book that the average energy requirement for humans to work is 2MJ per person per day, and that this produces 400kJ of useful work per day, making human being's energy-to-exergy efficiency only 20%. The World Bank states that the current labour force consists of 3.7 billion people, meaning that the average energy requirement of labour (assuming a 6-day work week) equals 2MJ × 3 billion × 312 = 1,872 billion MJ which equals 0.52 TWh per year. Primary energy demand in 2024 was 167584 TWh, meaning that labour's contribution to that was 0.52 TWh / 167584 TWh = 0.0003%.

necessary to either become more efficient at converting stored energy into usable energy, reducing energy losses, or to increase the overall capacity of capital to use energy, which requires additional energy to be available. The steam engine might again provide a good example of this.

Adding more engines with a fixed coal supply will simply reduce the performance of each individual engine. Improvements in the cylinder can increase the energy-to-exergy conversion of a given engine, but the efficiency is bounded between greater than 0 and less than 1 and it increases incrementally. Great technological shifts, such as moving from steam engines to internal combustion engines, not only improved upon the efficiency in conversion (still only ranging between 20 to 30 percent) but also allowed the engine to consume larger amounts of energy, increasing the overall energy throughput (and thus exergy output) of capital. These improvements can also be seen in terms of total factor productivity. Since a large part of growth is no longer unexplained, TFP is no longer an exogenous parameter, but can be reconceptualized as improvements in energy efficiency $e_{x,t}$ and/or in the energy throughput of capital $E_{K,t}$.

Lastly, as output in the above function is expressed in terms of usable energy, it is necessary to then convert this to a monetary metric from which consumption and investment are derived. This is done by dividing output by a conversion parameter, η . The exact calibration of this parameter can be found under Section 6.2. It is assumed to be constant, which gives:

$$GDP_t = \frac{Y_t}{\eta} \tag{11}$$

5.3 Low-carbon energy production with mineral constraints

Following the introduction of the new production function, the mineral supply is added as a constraint to low-carbon energy production. Similar to Chazel et al. (2023), low-carbon energy is produced from the stock of green capital. However, there is an important deviation in the pathway through which green capital is created. Producing low-carbon energy depends first on primary mineral extraction, contributing to the production of green capital, which in turn builds the green capital stock, and leads to the production of low-carbon energy.

First, a flow of primary minerals $m_{p,t}$ is extracted from the mineral stock $M_{p,t}$ each period. This is in line with how oil is conceptualized by Golosov et al. (2014) and based on the same notion that in terms of extraction costs, labour costs are assumed to be negligible with respect to the scarcity costs of minerals. Primary mineral extraction is bounded by the mineral reserves. This gives:

$$m_{p,t} = M_{p,t} - M_{p,t+1} \tag{12}$$

$$0 \leq m_{p,t} \leq M_{p,t} \tag{13}$$

Where the production of minerals depends solely on its scarcity, the production - or flow - of green capital does depend greatly on labour inputs. The labour share directly involved in green capital production is $L_{g,t}$, where g denotes the green capital production sector. Parameter $A_{g,t}$ exogenously measures labour productivity. I follow Chazel in using a standard CES production function with a negative parameter of substitution $\ddot{\rho}$, implying complementarity between labour and minerals.

$$green_t = \left[\kappa_{L,t}(A_{g,t}L_{g,t})^{\ddot{\rho}} + \kappa_{M,t}(m_{p,t})^{\ddot{\rho}}\right]^{\frac{1}{\ddot{\rho}}} \tag{14}$$

Here, $\kappa_{L,t}$ and $\kappa_{M,t}$ are the share parameters for labour and minerals, respectively. Where Chazel et al. (2023) explicitly model a secondary mineral sector, the current model implicitly incorporates different avenues of technological change by allowing these share parameters to change over time. This mechanism works as follows: Following Chazel's assumption that the production of secondary minerals depends only on labour, increased recycling implies that the relative share of labour would go up compared to that of primary minerals. This is because each unit of green capital would require relatively more labour than primary minerals. This mechanism then implicitly reflects a shift towards a technology using more recycled minerals or other more labour-intensive technologies, which would lead to an increase in the demand for labour in the secondary mineral sector, reducing the demand for primary minerals relative to labour to produce green capital. This demand, however, can never go negative, meaning that contrary to Chazel et al. (2023), a 100% recycling rate is not possible in this model.

The flow of green capital builds up the stock of green capital each period (think solar panels, wind turbines, etc.) which depreciates at a rate of δ_G:

$$G_{t+1} = green_t + (1 - \delta_G)G_t \tag{15}$$

The above functions for mineral extraction, green capital production, and the green capital stock are all expressed in terms of mineral quantity (megatonnes of copper in the calibration). Low-carbon energy is then produced from the stock of green capital, where it is multiplied by a conversion factor, ψ, to transform the stock of green capital (in Mt of minerals) into energy (in terawatt hours):

$$E_{3,t} = \psi G_t \tag{16}$$

In line with Chazel et al. (2023) it is thus assumed that there is a known and finite stock of minerals allocated to the energy transition (similarly to the oil stock in GHKT). This conceptualizes low-carbon energy as another nonrenewable resource, departing from Golosov et al. (2014) who assume that low-carbon energy can be generated indefinitely without constraints. As in GHKT and Chazel, it is also assumed that labour can move freely between all sectors at all times, where the feasibility constraint on labour allocation is:

$$L_{0,t} + L_{2,t} + L_{3,t} + L_{g,t} = L_t \tag{17}$$

6 Simulations

This section details the method applied for the model simulations. Section 4.1 briefly highlights the optimization algorithm, and Section 4.2 describes the (re)calibration of the model.

6.1 Algorithm

The simulation is run on MATLAB. The model optimizes over 30 time periods and five decision variables: the savings rate K_{t+1}, the remaining oil stock $R_{1,t}$, the remaining mineral stock $M_{p,t}$, and the labour shares for coal, $A_{2,t}$ and green capital production ,$A_{3,t}$. The *fmincon* solver is used with the interior-point algorithm, to maximize the sum of discounted value of utility.

6.2 Calibration

There are several changes to the way the model is calibrated compared to Golosov et al. (2014) and Chazel et al. (2023). The section below describes the justification for changing certain parameters and model units. Other parameters that remain unchanged have been omitted from this section. A full list of parameters can be found in the Appendix under Section A.

Normalizing to units of energy

An important change to the model, beyond the change in the production function and the addition of mineral constraints, is that most units are expressed in terms of energy. Departing from Golosov et al. (2014) who normalize and calibrate their model to gigatonnes of carbon (GtC), the current model is calibrated and normalized to units of energy, expressed in terawatt hours (TWh). This is an important change, and it is done for three reasons.

The primary reason is that since energy is the main focus of this thesis, working with units of energy eases the interpretation of important variables such as oil, coal, and low-carbon energy production. Normalizing to GtC makes sense from a conceptual standpoint, as it is in line with GHKT's aim of identifying pathways for resource extraction under optimal carbon taxation. However, their choice also makes their calibration process quite opaque. For example, they assume unity for the relative price per unit of carbon between oil and renewables, and as the production of each energy source is expressed in GtC. This makes sense for oil and coal but is difficult to interpret for low-carbon energy (technically peaking at 966 GtC by the end of their optimization period - nearly 10 times the current amount of global annual emissions). The reconfiguration to units of energy therefore makes the outcome easier to interpret at every step of the optimization process, where the production of oil, coal, and low-carbon energy are immediately expressed in terms of energy.

Second, normalizing to units of energy has the added benefit of reducing the number of conversions needed to calculate output, as the new production function requires that inputs into the production function are units of energy (Keen et al., 2019). Lastly, perhaps independent of the exact units used, recalibration is necessary in general as the state of energy production has changed massively since 2014. For instance, the price of electricity from solar energy dropped by 88% over the last 15 years, making it the cheapest source of electricity together with onshore wind energy (Roser, 2020). The recalibration of important parameters will better reflect these new dynamics.

Share parameters of energy

The new share parameters are calibrated based on the relative prices of different energy sources. A straightforward comparison between these is difficult to achieve, not only because of the heterogeneous nature of low-carbon energy technologies, but also because fossil fuels are often measured in different units and differ in important features such as energy density, storage capability, and end uses. Departing from Golosov et al. (2014), who take the relative price in terms of dollars per ton of carbon, I normalize energy sources based on their price per MWh. To do so, I take the levelized cost of energy (LCOE) for coal, gas, and renewables. The LCOE captures the cost of building the power plant as well as the ongoing costs for fuel and operating the power plant over its lifetime, without subsidies. It provides insights into the cost competitiveness of various energy generation technologies and thus offers a tool for more consistent comparisons between energy sources. In 2024, the LCOE was \$30 for onshore wind, \$40 for solar photovoltaic, and \$115 for coal, all per megawatt hour (Roser, 2020).

The comparison with oil is less straightforward, as oil is not generally used in power generation, but rather as a fuel. To apply the same rationale as the LCOE, I will calculate the relative price of oil by using the energy content of gasoline, which will approximately account for factors such as energy losses and production costs. This starts with the average crude oil price of \$81 per barrel in 2024 (EIA, 2025). As 1 barrel of oil on average contains 20 gallons of gasoline, and each gallon on average has a stored energy density of 33.41 kWh, one barrel of oil contains 668.2 kWh of stored energy, giving a relative price of (81/0.6682=) \$121.22 MWh.

To arrive at the share parameters, I use data on the change in global primary energy consumption from the Statistical Review of World Energy for the years 2014-2024 as provided by Our World in Data, see Ritchie et al. (2023). This is done to take into account the lag between low-carbon energy price decreases and low-carbon energy production increases. Looking purely at annual consumption, rather than the change in consumption over the last decade, would risk underestimating the share parameter of low-carbon energy in the energy production function. This data shows a change in consumption of 4955.69 TWh for oil, 938.17 TWh for coal, and 5074.32 TWh for low-carbon energy. Taking these updated consumption levels and relative prices, and using the existing equation from Golosov gives:

$$\frac{\kappa_1}{\kappa_2}\left(\frac{E_{1,t}}{E_{2,t}}\right)^{\rho-1} \quad \text{and} \quad \frac{\kappa_1}{1-\kappa_1-\kappa_2}\left(\frac{E_{1,t}}{E_{3,t}}\right)^{\rho-1} \tag{18}$$

Taking the parameter of substitution directly from Golosov, ρ = -0.058, which corresponds to an elasticity of substitution of 0.945, the new share parameters are $\kappa_1 = = 0.455$, $\kappa_2 = 0.078$, and $\kappa_3 = 1 - \kappa_1 - \kappa_2 = 0.467$. These slightly deviate from Golosov, in part because they calibrate their share parameter for oil by using the combined demand for oil and gas, increasing its consumption by more than 50%. Their share parameters are $\kappa_1 = 0.5429$, $\kappa_2 = 0.1015$, and $\kappa_3 = 0.3556$. This means in the current calibration, oil and coal become relatively slightly less important in the production of energy, and low-carbon energy slightly more.

The oil stock

For the oil stock, current proven oil reserves estimates range from 1500 to 1700 billion barrels as of 2024. Taking 1600 billion barrels as a starting point, and given its energy content of 1700 kWh per barrel, yields $1{,}600 \times 10^9 \times 1{,}700 = 2.72 \times 10^{15}$ kWh = 2,720,000 TWh. Hence, the oil stock is taken to be 2,720 x1000TWh.

Low-carbon energy production

In the model, the production of low-carbon energy follows four distinct steps. First, minerals are extracted from the mineral stock. This stock is estimated to be 2000 megatonnes of copper (MtCu) in line with the upper estimate by Chazel et al. (2023). In Section 8 the impact of lower estimates of the copper stocks are also analyzed. Second, these minerals are then inputs to the production of green capital, together with labour. The relative efficiency of primary minerals in this production process is initially taken to be 0.75 which is also taken directly from Chazel et al. (2023). In the baseline model this share is fixed, but an important departure from Chazel is that I allow this relative efficiency to change over time, as explained in Section 5.3.

Third, the production of green capital then adds to the stock of green capital. The initial stock of green capital is taken to be equal to 53.8 MtCu, calculated from the 2021 annual supply of copper to clean energy technologies, which was 5380 kilotonnes (IEA, 2024a). Multiplying this by 10 for the decadal stock gives 53800 kilotonnes, which equals 53.8 MtCu. Low-carbon energy flows from this stock of green capital. The fourth and last step is the conversion of green capital, in MtCu, to low-carbon energy, in TWh. This is calibrated by dividing the annual amount of low-carbon energy production by the copper used in clean technologies that year. For 2024 this gives 9225 TWh/6.311MtCu = 1462.5 TWh per MtCu, expressing this in the model units gives 1.46 x1000 TWh/MtCu. This means that 1 Mt of copper embedded in low-carbon technologies corresponds to 1460 TWh of low-carbon energy production.

Labour productivities

Recalibration is also necessary for the labour productivities in the coal and low-carbon energy sector, which should now reflect their contributions in terms of energy. The labour productivity of coal, $A_{2,0}$, is calibrated using updated average extraction costs of \$65 per ton (IEA, 2024a). Applying the same formula as Golosov et al. (2014), where the labour supply is normalized to unity and the share of world labour used in coal extraction is assumed to be zero, the wage is given by $w_t = (1-\alpha)Y_t$. Using the world GDP of \$1100 trillion per decade, and a price of \$65 per MWh the cost of in TWh of energy is:

$$A_{2,0} = \frac{(1-\alpha)Y_0}{p/_{\mathrm{TWh}}} = \frac{0.66 \cdot 1100 \cdot 10^{12}}{65 \cdot 10^{6}} \approx 11,169,231 \tag{19}$$

Meaning to extract 1000 TWh of energy in the form of coal, a share of $\frac{1}{11,169}$ of the labour supply of a decade is needed.[12] The calibration of $A_{3,0}$ is then derived by noting that $A_{3,0}/A_{2,0}$ is equal to the relative price between coal and low-carbon energy, which gives $A_{3,0} = 11{,}169.231 \times 35/115 = 3{,}399.331$ x1000 TWh. A second departure from Golosov et al. (2014) in terms of labour productivity concerns the assumptions on productivity growth in the energy sectors. Where GHKT assumes annual labour productivity increases by 2% annually for both the coal and the low-carbon energy sector, I assume productivity in the coal sector increases by 1% annually, meaning productivity growth in the low-carbon sector outpaces that of coal, better reflecting historical learning rates (Roser, 2020).

[12] This was initially calibrated using $1-\alpha-\upsilon = 0.66$. With the current change to the production function this should have been $1-\alpha = 0.67$, which would lead to a slightly higher productivity. However, this is not expected to change model outcomes, and the ratio between coal and low-carbon energy production remains the same.

Calculating emissions

The side effect of this recalibration is that the conversion to emissions becomes slightly less straightforward. Given that now the units of energy produced are in TWh, to arrive at emissions, it is necessary to trace back the units of energy to physical units of oil and coal. Following Golosov et al. (2014), one metric ton of crude oil contains 7.33 barrels (or 1 barrel contains 0.136 metric tons) and 1700 kWh of stored energy. With a carbon content of 846KgC per ton of oil, this means there is 0.0676 GtC per thousand TWh. Doing the same for coal, and assuming that 1 ton of coal contains around 7.11 MWh of stored energy, gives around 0.1008 GtC per thousand TWh. Applying these carbon emission factors to each respective energy production function gives the amount of total emissions in GtC, which is used to calculate cumulative carbon emissions and the associated climate damages.

From energy to GDP

Lastly, as output is expressed in terms of usable energy, it is necessary to convert this into a measure of GDP. This is done by first calibrating the usable energy throughput of capital, which is necessary to make an estimation of initial output, which, divided by observed GDP, provides a conversion factor for translating output (in x1000 TWh) into GDP (in billion dollars).

The initial average usable energy throughput of capital $E_{K,0}$ is equal to the initial usable energy supply, divided by the initial capital stock:

$$E_{K,0} = \frac{e_{x,0} \cdot E_0}{K_0} \tag{20}$$

To calibrate this, I take the initial energy-to-exergy efficiency, $e_{x,0}$, to be 33%. It is difficult to find an aggregate measure of energy-to-exergy efficiency in the economy, as conversion efficiencies differ between technologies and over time. However, given that conversion efficiencies of coal plants are around 30-40%, while wind turbines and solar panels range between 20-40%, the assumption that, on average, one-third of primary energy is converted into useful exergy available for production seems reasonable (Khawaja et al., 2024).

Initial energy production, E_0, is then estimated based on the CES production function for energy (see Equation 6) with the parameter of substitution, $\rho =$ -0.058. The share parameters are taken from Equation 18 above, and 2024 energy consumption from Ritchie et al. (2023). Expressed in the model units of one thousand TWh, energy demand was 55.292 for oil, 45.851 for coal, and 9.225 for low-carbon energy. Lastly, the estimation of the initial capital stock is calibrated following Golosov et al. (2014), and expressed in billion US dollars. This gives $E_{K,0} = 0.00003418$.

Once the usable energy throughput of capital has been calibrated, this enables an estimation of initial output, Y_0. For simplicity I take the labour supply to final goods production to be 1, assuming that the labour share going to the energy sectors is negligible,. This simplifies the production function of Equation 10. The conversion parameter, η, is then calibrated based on an estimation of initial output Y_0 (measured in units of usable energy) divided by observed GDP (in dollars).

$$\eta = \frac{Y_0}{10 \cdot GDP_{2024}} \tag{21}$$

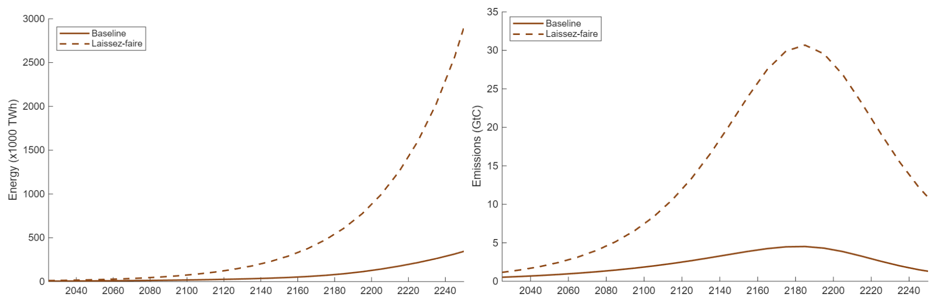
This parameter captures the amount of usable energy in one thousand TWh embedded in one dollar of GDP in the model. Given that Y_0 in the model is estimated to be 4.17 per decade, and observed GDP in 2024 was 110,000 billion dollars, $\eta = 4.17 / 1100000 = 0.0000038$. All parameter values can be found in the Appendix.

7 Results

This section presents the results of the model simulations. In the baseline scenario, described in Section 7.1 , the new production function and mineral resource constraints are implemented, and outcomes are compared with the optimal carbon tax (the baseline scenario) or without the optimal carbon tax (the laissez-faire scenario). The results of these two are discussed in terms of fossil fuel use, carbon emissions, temperature, and the carbon tax. A comparison will be drawn with the benchmark model from Golosov et al. (2014), however, it is important to note that initial values are not yet calibrated correctly, which means that the discussion will mainly concern different trajectories, and not magnitudes (an issue which is discussed in depth under the Limitations in Section 9). Section 7.2 then takes the baseline model, and zooms into the role of technological innovation in both the production of low-carbon energy and the production of final goods. Lastly, by zooming into the differences between capital-directed and energy-directed technical change, Section 7.3 discusses the implications of technology on long-run economic growth. A full description of the different scenarios and their calibration can be found under the Appendix Table 4. Overall, the results highlight that assumptions on technology matter greatly for both the low-carbon energy sector and the economy as a whole.

7.1 The baseline

The baseline model below integrates the new production function and mineral constraints to low-carbon energy production, and it aims to stay close to the original GHKT assumptions. Most importantly, the baseline scenario assumes that capital and green capital fully depreciate over the course of one period,



(a) Production

(b) Emissions

Figure 2: The coal sector

also in line with Chazel et al. (2023), and that the only channel for productivity growth is through labour productivity increases in the coal and low-carbon sector. Compared to GHKT, the model finds different lower optimal paths of energy use, emissions, and carbon taxation.

Fossil fuel production and carbon emissions

The trajectory of coal production changes depending on whether the carbon tax is implemented or not. As expected, if there is no penalty for emitting carbon into the atmosphere, coal production increases compared to the optimal scenario. Figure 2a depicts the coal sector's decadal production levels. The predicted quantities are likely incorrect and off by one magnitude; current primary energy demand for coal is 55,000 TWh annually where the model predicts 5,200TWh per decade. Nevertheless, some important dynamics from Golosov et al. (2014) remain.

The implementation of the carbon tax immediately reduces coal use by around 55% (compared to 46% in GHKT) and this increases to a 5-fold difference after 100 years (compared to a 7-fold difference in GHKT, as their use of coal grows quicker under laissez-faire). The model also keeps the assumption that a backstop technology emerges, after which emissions from coal smoothly decline to zero (see Appendix Figure A.1). The right panel shows that without a carbon tax, this leads to a peak in emissions from coal before an eventual decline once the emission coefficient decreases. This peak in emissions is much flatter in the optimal scenario. Despite a discrepancy in predicted quantities, the main message is shared with Golosov et al. (2014): Coal use is the main driver of emissions, and thus, under the optimal carbon tax, coal production should be reduced.

Similar to GHKT, there is almost no variation in oil production levels between the optimal and the laissez-faire scenario (not depicted here, see Appendix Figure A.2). The most notable difference is that the current model finds that optimal oil use is constant around 82,000 TWh per decade, which would come down to 5.6 GtC per decade, whereas Golosov et al. (2014) predict a steady decline in oil from 32 GtC to less than 5 GtC per decade. This constant trajectory contradicts what one would expect under Hotelling's rule, but it can be explained by calibration of the baseline model where energy is abundantly available despite its constraints, making that the scarcity of energy is not binding, and that the marginal product of oil is roughly equal to zero. More attention is paid to this issue under Section 7.3.

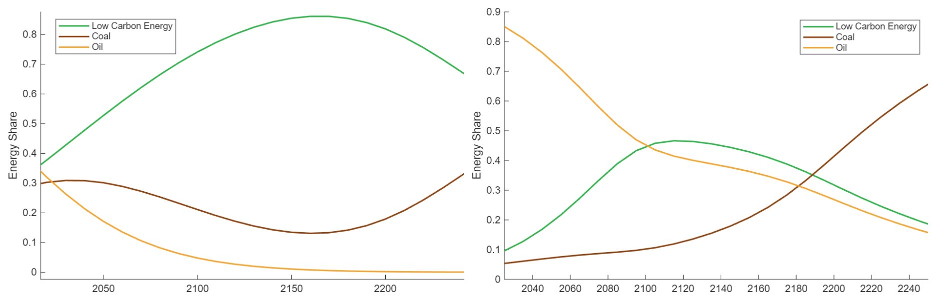
The relative shares of each energy source in the total production of energy are depicted in Figure 3. As can be seen in Figure 3a in the optimal scenario for GHKT, the use of coal is favoured over oil almost immediately, and oil supplies less than 10% of all energy by the end of this century. Low-carbon energy then compensates for the declining oil supply, where the limitless availability of minerals enables a peak of nearly 90% of total energy.[13] The relative contribution of low-carbon energy declines once coal production becomes cleaner.

Figure 3b shows a different trajectory under the baseline scenario, where each energy source is constrained by its emissions, scarcity, or both. Oil initially supplies nearly 90% of all energy (likely because the initial quantities of the other energy sources are not yet calibrated correctly) but its share immediately declines as low-carbon energy production is scaled up. In contrast with GHKT, oil remains an important source of energy throughout, still supplying 20% of all energy after 200 years. Around 2100, as oil and minerals become more scarce, coal emissions start declining rapidly, increasing its importance in the energy mix. The peak in the relative importance of low-carbon energy is thus lower than in GHKT, and it is overtaken by coal as the main source of energy sooner. Again, despite discrepancies in the calibration of initial quantities, the changes in trajectories are in line with what one would expect considering the introduction of mineral constraints. As low-carbon energy is now also linked to nonrenewable resource use, it cannot be used as freely as before. This makes oil use more favourable in the early phases, while coal is still very polluting, and it increases the need for coal once oil and minerals are reaching depletion. These new constraints on energy then also impact pathways for economic growth and carbon taxation.

Carbon taxation and climate change

By definition of the model specification, the carbon tax equals a constant share of output, and this optimal share only depends on only three parameters: the discount rate, the expected damage elasticity, and the structure of carbon depreciation in the atmosphere. As none of these are changed in the current model, the carbon tax to GDP ratio is the same between the GHKT model and the baseline model, at 0.000087% of GDP. When expressed in dollars, this equals $0.161 per ton of carbon, which is significantly lower than Golosov's optimal tax, ranging between $32 and $56. This relates to the reconfiguration of the

[13]To achieve this, nearly 2% of the total labour force is dedicated to low-carbon energy production in the first decade (see Appendix Figure A.4).



(a) Golosov et al. (2014) (b) Baseline

Figure 3: Energy mix: GHKT & Baseline

production function, where further constraints are imposed on energy, reducing the need for carbon taxation as an instrument to reduce emissions.

The tax is computed based on the present discounted value of the marginal climate damages caused by an additional unit of carbon emissions, weighted by the marginal utility of consumption over time. In essence, the tax internalizes the cost of emissions that would otherwise reduce future welfare. In the current model, the central role of energy in production already imposes a constraint on emissions because energy use cannot increase as freely, which limits production. The difference between emissions between the optimal tax and the laissez-faire scenario is much lower in the current model than in GHKT. Even in the laissez-faire scenario, emissions never rise as high as they would under the optimal scenario for GHKT. Driven by coal use, and despite the declining emission intensity, their emissions rise near exponentially towards the end of the optimization period (See Appendix Figure A.3). As a result, in the baseline scenario the tax is less ”necessary” to keep emissions in check, resulting in a markedly lower carbon tax.[14]

Figure 4 illustrates the impact this has on temperature. The current model starts at a warming level of 1.39°C in 2024, which closely approximates observed levels of global warming.[15] Even in the unconstrained scenario, global warming peaks around 2°C, and with the optimal tax it flattens around 1.6°C. Golosov et al. (2014) find a much larger increase in global temperatures of 3.4°C compared to preindustrial levels, even in their optimal scenario, which explains why their carbon tax is so much higher.

Overall, these results align with expectations. The optimal carbon tax reduces

[14]The finding of a relatively low optimal carbon tax is consistent with results elsewhere in the literature, though for different theoretical reasons, by highlighting the importance of the elasticity of substitution between “dirty” and “clean” energy sources (Acemoglu et al., 2012). This is elaborated on in Section 9.

[15]Copernicus finds a level of 1.6°C warming in 2024 alone, but 1.35°C when taking into account a five-year average, see Forster et al. (2025).

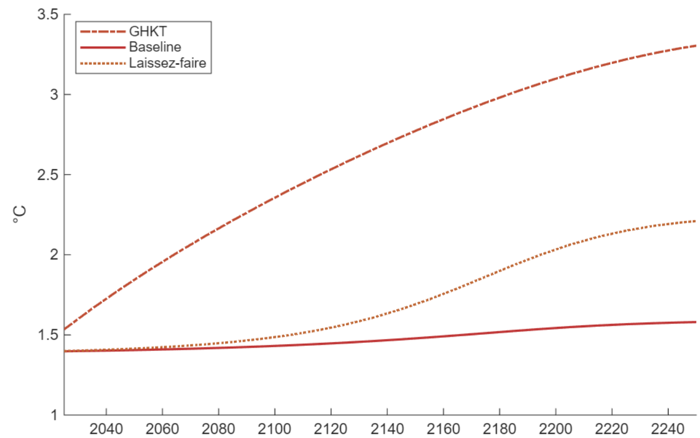


Figure 4: Temperature Increase

emissions compared to the unconstrained scenario. These emissions are mainly driven by coal production, meaning that while the sector is emission-intensive, coal use should be reduced. Furthermore, implementing mineral constraints makes oil relatively more important in the energy mix, reducing the peak of low-carbon energy to less than 50% of the energy supply. Even without carbon taxation, the new role of energy in production means that climate damages are much lower compared to GHKT. This is also linked to the role of energy in GDP, but before exploring this in Section 7.3, the following section will analyze the impact of changes to important technological parameters on the low-carbon energy sector and the production of final goods.

7.2 The impacts of technological progress

The above results come from a model with relatively restrictive assumptions on technological progress. The following section will analyze the extent to which outcomes change when changing the underlying assumptions on technology by examining four distinct channels. The first two channels relate to innovation in the low-carbon energy sector: a slower depreciation of green capital, and a reduction in the relative efficiency of minerals in the production of low-carbon energy. The last two channels relate to the role of energy in final goods production: producing energy more efficiently and increasing the capacity of capital to harness energy. Each change is presented as a different "scenario", and a full overview of these scenarios and their specifications can be found in Appendix Table 2.

Innovation in the low-carbon energy sector

Table 1 shows different assumptions on technological parameters for 5 different scenarios. In the baseline scenario, green capital fully depreciates over the course of one decade, and the share parameter of minerals, κ_M, is constant. However, it is reasonable to assume that, over time, primary minerals become relatively less important in the production of green capital. Scenario 1 therefore models a decline of its relative efficiency of 0.2% per year, meaning that after a period of 100 years, minerals almost become an order of magnitude less important in the production of green capital, relative to labour.[16] This is can be due to increased recycling, in line with Chazel et al. (2023) who find that green capital is fully made up of recycled minerals after 100 years, but it can also reflect other structural changes, such as the growing role of skilled labour in the production, construction, and maintenance of low-carbon energy technologies.

A second important channel through which innovation might reduce the impacts of increasing scarcity is through extending the lifespan of green capital, δ_G. In Scenario 2, green capital depreciates more slowly, at a rate of 10% per year. This better approaches observed depreciation rates, as the current lifespan of wind turbines, for instance, is between 25 and 30 years. Scenario 3 combines these innovations in the depreciation and relative efficiency of minerals. Finally, Scenario 4 models the impact of different mineral stocks to see if these innovations can offset the impact of increased mineral scarcity.

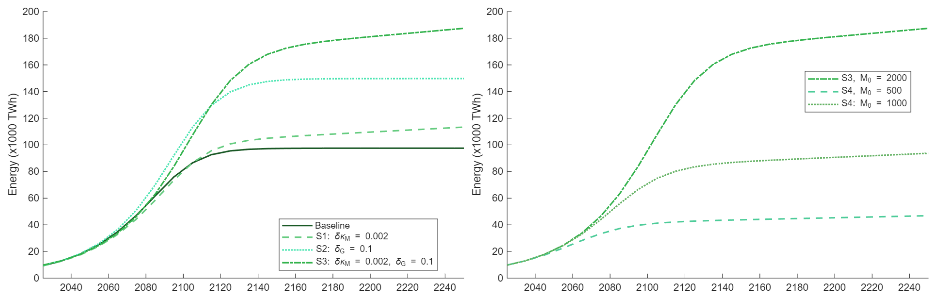
Table 1: Innovation in the production of low-carbon energy

	$\delta\kappa_M$	δ_G	M_0
Baseline	-	1	2000
Scenario 1	0.2%	1	2000
Scenario 2	-	10%	2000
Scenario 3	0.2%	10%	2000
Scenario 4	0.2%	10%	[500, 1000]

Figure 5a shows the development of the low-carbon energy sector under the baseline and Scenarios 1, 2, and 3, leaving the initial mineral stock unchanged. Similar to the findings by Chazel et al. (2023), there are only small differences between the four scenarios in roughly the first 50 years, and then their trajectories diverge.[17] As the baseline scenario assumes the only increase in productivity in the low-carbon sector comes from labour productivity increases of 2% per year, low-carbon energy production reaches a plateau after around one century. Chazel et al. (2023) also find a plateau, although in their scenario with the same

[16]This may seem small, but despite its high physical recyclability, the input rate of recycled copper has remained constant over the last 15 years (IEA, 2024b). Given that the IEA projects a supply shortage in the coming decade, it is likely that primary mineral extraction costs will become relatively more important in production, making a decline of 0.2% per year a relatively optimistic assumption.

[17]To prevent a large decline in the green capital stock after the first period, the initial green capital stock is recalibrated to be lower under the non-baseline scenarios.



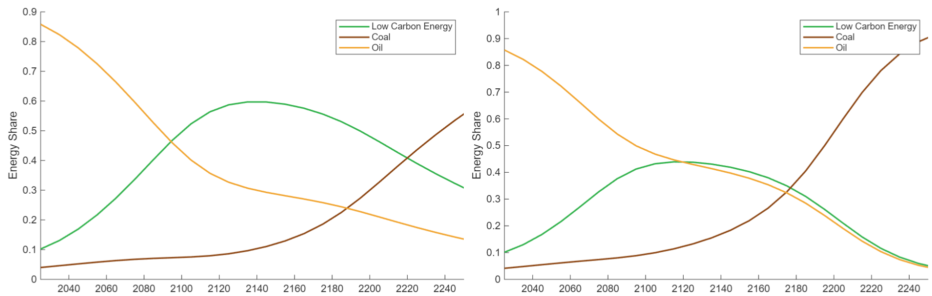
(a) Technological progress (S1, S2, S3) (b) Changing mineral stocks (S4)

Figure 5: Low-carbon energy sector

mineral stock, this plateau is reached some 50 years later. Furthermore, where they see an increase in copper extraction from 50 MtCu to 80 MtCu in the first century, copper extraction under the baseline model is constant at around 60 MtCu per decade for the whole optimization period. This matches the current level of copper extraction of 6 MtCu per year (IEA, 2024a).

In Scenario 1, on top of labour becoming more productive, primary minerals become relatively less important. As a result, production does not reach a plateau but instead maintains a small positive level of growth. In Scenario 2, there are no improvements in the relative efficiency of minerals, but the lifespan of green capital is extended to depreciate around 65% per decade. Here, the peak of low-carbon energy production is reached around two decades later and at a higher level of production, at nearly 150,000 TWh per decade compared to 95,000 TWh in the baseline. Scenario 3 then combines both pathways. With slower depreciation and a declining relative necessity for primary minerals, the sector can produce a higher amount of low-carbon energy, and production levels off around two to four decades later than with no technological improvements.

A final important parameter for the development of the low-carbon sector is the estimated mineral stock available to the energy transition, M_0. Scenario 1, 2, and 3, all assumed a mineral stock of 2000 MtCu, in line with the upper estimates of Chazel et al. (2023) and more than twice as large as the current amount of estimated copper reserves. Figure 5b shows that in Scenario 4, despite the improvements in terms of efficiency and depreciation of minerals, a lower mineral stock of 1000 MtCu or 500 MtCu greatly reduces the supply of low-carbon energy. Figure 6 shows the impact this has on the energy mix over time. Figure 6a illustrates how in Scenario 3, technological innovation supports higher levels low-carbon energy production, which raises its share in the energy mix and reduces that of coal compared to baseline in Figure 3b. However, this increase is not maintained with a smaller mineral stock. In Figure 6b, a smaller mineral stock lowers the contribution of low-carbon energy to the mix over time, meaning the onset of coal happens sooner. This suggests that the benefits of technological innovation may not offset the impact of increasing scarcity.



(a) S3: $M_0 = 2000MtCu$

(b) S4: $M_0 = 1000MtCu$

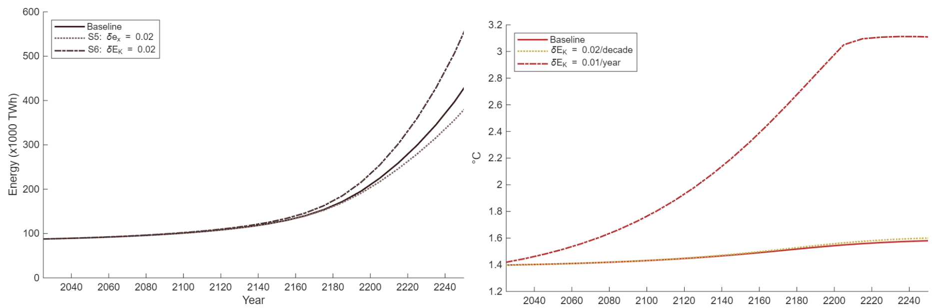
Figure 6: Energy mix with $\delta\kappa_M = 0.002$ & $\delta_G = 0.1$

These results therefore replicate some important dynamics from Chazel et al. (2023): (i) increased recycling delays the peak of low-carbon energy production, (ii) and a longer life span allows the production of a greater amount of low-carbon energy. In terms of timing, the results are comparable as well. Chazel et al. (2023) find that increased recycling delays the plateau of low-carbon energy production by 40 to 60 years, depending on their assumptions on the mineral stock. In the current model, this is slightly sooner, by around 20 to 40 years. Additionally, they find that recycling 'significantly' changes the trajectory of the sector only after around 80 years. This is the same in the current model. Lastly, technological improvements increase the share of low-carbon energy in the energy mix and delay the onset of coal production. 6b shows a lower mineral stock means relatively more oil is used for a longer period of time, a finding similar to Fabre et al. (2020). These results are all quite intuitive and in line with expectations.

Interestingly, where the different pathways for the production of low-carbon energy do impact the aggregate energy supply (see Appendix Figure A.5), the increase in energy has little to no impact on the trajectory of economic growth. The reason for this relates again to the specification of the new production function, where energy and capital are perfect complements, meaning that when capital is the limiting factor, increases in the usable energy supply do not directly translate into increased output. The following section will analyze this relationship.

Capital and energy-directed technical change

Instead of modeling technologies specific to the low-carbon energy sector, another pathway of innovation is through changing the use of energy in producing final output. As discussed in the theoretical framework, technological progress can be directed towards improving the productivity of capital, or towards using energy more efficiently - reducing energy losses. In the model, these are



(a) Fossil fuel use (S5, S6) (b) Temperature change (S5)

Figure 7: Capital-directed technological change

captured by changes to the terms E_K, the average energy throughput of capital, and e_x, the energy-to-exergy efficiency of energy in final goods production. Where energy-directed change reduces the average energy requirements per unit of output, capital-directed technical change is energy-biased, which means that it increases the average energy requirement per unit of output.

These ideas are reflected in Figure 7a. The baseline model reflects the use of fossil fuels without growth in E_K and e_x. In Scenario 5, when allowing the energy-to-exergy efficiency to grow at a rate of 2% per decade - meaning it will grow from 0.33 to 0.59 over the optimization period - there is a small decline in the use of fossil fuels compared to the baseline. When instead allowing the energy throughput of capital to grow by 2% per decade in Scenario 6, there is an upward shift in fossil fuel usage.

In Figure 7b, the impact of this becomes more evident when allowing the energy throughput of capital to increase faster, by 1% per year, more in line with current trends of global energy consumption increasing by 1% to 2% annually. A stronger increase in the energy throughput of capital drastically increases global warming by increasing fossil fuel demand and emissions. Basically, the growing "energy-hungriness" of capital does not distinguish between energy sources, and with no efficiency improvements in the low-carbon sector, this increased demand for energy is met by fossil fuels. Overall, these findings suggest that indeed, capital-directed technical change has an energy bias. This increase in the capacity of capital to harness energy also has implications for economic growth.

7.3 Implications for long-run economic growth

The reconfiguration of the production function has important implications for economic growth. Under the baseline specification with full depreciation and limited productivity increases, the results show an initial decline in GDP, as opposed to GHKT, who find an initial increase. However, both models eventually find a small but steady declining trajectory. From 2075 to 2225, GDP declines

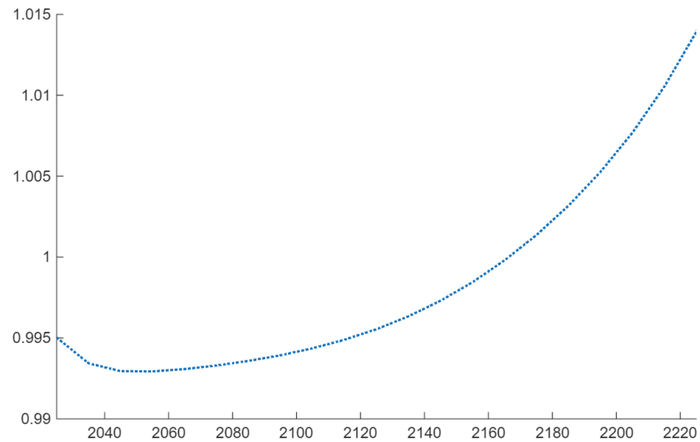


Figure 8: Net output: baseline versus laissez-faire

with 1.5% in the GHKT model and with 0.9% in the baseline. The initial values are not yet calibrated correctly, which is why the development of GDP in dollars can be found under Appendix Figure A.6. In terms of net-of-damages output, Figure 8 shows the ratio between output under optimum or under laissez-faire, GHKT finds a small loss in the first period but a 2.5% higher GDP after 100 years. In the current model, these differences are smaller. Optimal GDP is still 0.5% lower after 100 years, and only 1.5% higher after 200 years.

Figure 9 shows the development of economic growth relative to the baseline for different assumptions on technological change, each represented as a different scenario. It is immediately clear that changes in the usable energy throughput of capital, E_K, have the strongest impact on the trajectory for economic growth. Changing only the energy-to-exergy efficiency does not impact growth at all. This may seem strange, but it is again in line with the current specification of the model.

In terms of the production function, any instance where $E_K * K$ is larger than $e_x * E$ signifies an economy that is *energy-constrained*, because the ability of capital to harness energy is larger than the amount of useful energy being supplied. The reverse then signifies an economy that is *capital-constrained*. Because energy and capital are now perfect complements, growth happens by targeting the limiting factor. Despite the introduction of mineral constraints on energy, the current model is capital-constrained, meaning that in the complementary relationship, the limiting factor is capital - or the ability of capital to harness energy. Indeed, the ratio of capital's energy throughput and the usable energy supplied drops below one after the first period (see Appendix Figure A.7).

In a capital-constrained economy, energy is relatively cheap. As Acemoglu (2002) explained, when energy is relatively cheap, innovation is typically capital-directed. In the model, innovation that targets improvements in the usable en-

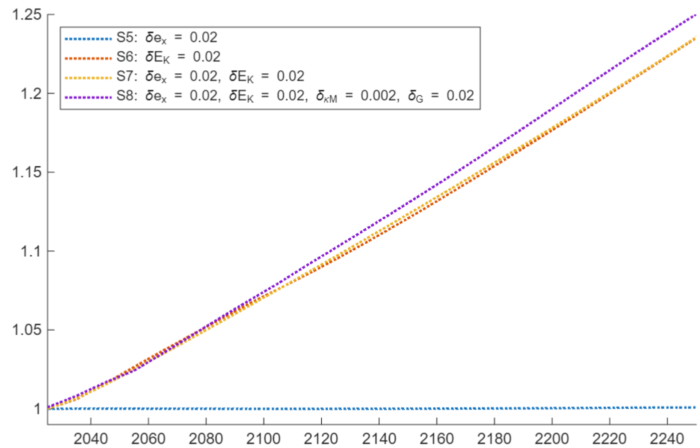


Figure 9: Net output with technological change

ergy supply are do not lead to growth without an associated change in capital. This explains why growth in e_x alone seems to have little impact on output, and growth in E_K matters a lot. Another avenue to increase output would be to increase the capital stock. But the current production function changes the pathway through which this stock is built up. Where in the Cobb-Douglas specification, energy contributes to the capital stock by independently contributing to output, the reformulation changes this dynamic. With capital being the limiting factor, adding more energy does now not increase output, meaning it does not build up capital. It also explains an early decline in output, as the initial capital stock is larger than what the economy can maintain given the initial energy supply. This means that the initial calibration conditions become hugely important, as they determine if the regime is capital or energy constrained. This is discussed further under the Limitations Section 9.

All in all, the results are in line with what theory would suggest. Innovation is important to offset the impact of resource scarcity, but the impact of innovation on growth depends on other factors. In the current regime, where energy is still relatively abundantly available at a price arguably far below its true cost, energy-directed changes do improve the energy efficiency of production, but they do not increase output. It simply lowers the energy intensity per unit of output. Bringing this down to a concrete example, a company that switches to more energy-efficient machines will reduce its overall energy costs, where each unit of output is simply produced with a lower energy intensity. This does not increase output by default. If the savings from reducing energy use are instead invested in an additional machine, more output can be produced, but the overall energy savings might be offset by the increase in the energy throughput of capital. Economy-wide, this might explain why, despite massive gains in efficiency, absolute demand for energy continues to grow: Growth in the energy throughput of capital outpaces growth in energy efficiency.

8 Sensitivity Analysis

In this final section, I take the baseline model specification and briefly test the sensitivity of GDP to changing four important parameters: the output elasticity of capital, α, the initial capital stock, K_0 , the discount rate, β, and the emission coefficient of coal, Υ.

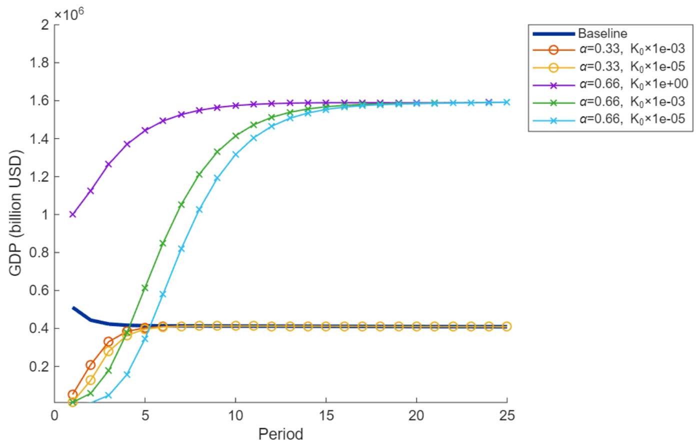


Figure 10: Sensitivity of GDP to α and K_0

Capital sensitivity

Under the Cobb-Douglas production function, the output elasticities of the inputs equal one, and this assumption is still present in the current production function, where the output elasticity of the capital-energy complement, α, is 0.33. The critiques against calibrating these output elasticities based on their cost shares have been discussed. Keen et al. (2019) argue instead for an output elasticity of the capital-energy term of 0.66. Applying this to the model increases the contribution of the energy productivity of capital to overall output. At the same time, assumptions on the initial capital stock also matter greatly for the initial levels of output. Testing the impact of calibrating this stock differently, by arbitrarily dividing the initial stock by 1000 or 100000, is aimed at illustrating the impact of starting in a different regime.

The impact of this change is immediately clear in Figure 10. In all specifications but the baseline, GDP shows some level of initial growth in GDP before an eventual stagnation. In the models where alpha is 0.33, the eventual plateau is the same as the baseline scenario, but initial GDP starts lower because the initial capital stocks are also reduced. While the capital stock is low, there is some room for growth, but this levels off after about 80 years.

Where the initial levels of GDP under a smaller capital stock are also lower than the baseline, changing the output elasticity of capital to 0.66 leads to a much

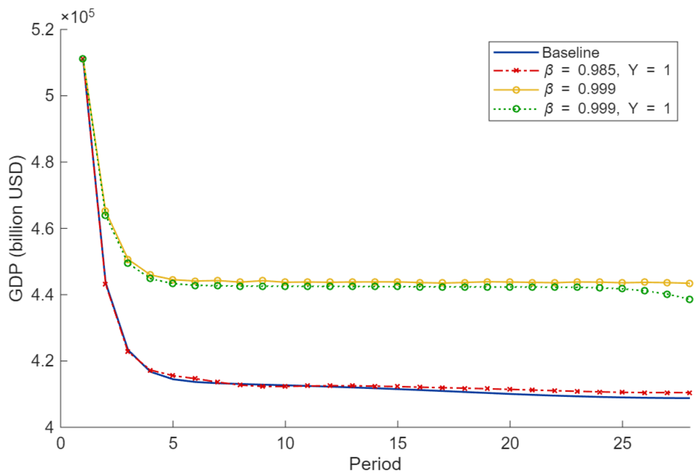


Figure 11: Sensitivity of GDP to β and Υ

steeper increase in economic growth. Now, the calibration of the initial capital stock puts the economy in an energy-constrained regime, where additional energy production can increase output by even more than before, which allows the economy to build up the capital stock. This is a virtuous cycle where more energy means more output, and more output means a larger capital stock. This effect also levels off, but much later than before. Even without a recalibration of the capital stock, the change in alpha means that initial output is much higher, meaning the capital stock is built up sooner, allowing the economy to escape the trap of a low capital stock, reaching higher levels of output.

Discounting and the emission coefficient

Two other important parameters are the discount rate, β, and the emission coefficient of coal, Υ. In the baseline specification, a discount rate of 1.5% per year means that future utility and climate damages are discounted by roughly 1.5% each year, implying that welfare losses or benefits occurring a century from now are valued at less than a quarter of their present value. This then relates to the emission coefficient of coal, which is modeled to smoothly decline to zero from around 2100 onward. In essence this makes coal a "clean" backstop technology that emerges in the future. Changing the coefficient will therefore increase emissions for a given amount of coal production, which could interact with the discount rate as higher future emissions will have a higher present weight.

Figure 11 shows the impact these different assumptions have on GDP. Overall it seems that the emission coefficient has little impact on the trajectory of GDP. With a lower discount rate, initial output still declines but plateaus at a higher level than under the baseline. The mechanism at work is that with a lower discount rate, there is less urgency to consume now, as the difference between current and future utility is smaller. The savings rate then increases from 28.4% in the baseline to 32.5% (the GHKT savings rate is 25.8%). More savings mean more capital is accumulated, softening the impact of the initial “capital trap” and enabling a higher level of GDP.

Across all specifications, the model shows similar dynamics to Golosov et al. (2014) but with important differences in scale and timing. Doubling the output elasticity of capital raises initial GDP before it eventually plateaus, resembling the general trajectory of GDP in the GHKT benchmark. A lower discount rate slows the initial decline in output, enabling a stabilization of GDP at a higher level than the baseline, which is in line with the sensitivity analysis for GHKT by Barrage (2014). In all cases, GDP ultimately levels off, reinforcing the idea that under the new production function, sustained growth requires additional usable energy to be available. The limitations and implications of these results will be further discussed below.

9 Discussion & Limitations

Limitations

Although much can be improved upon in the current model, there are two main limitations: The exact specification of the production function and the way it is currently calibrated. Regarding calibration, there are clear issues with the initial values of energy production, which are off by one magnitude. It seems the model predicts initial yearly consumption as decadal, for instance, for low-carbon at 9280 TWh per decade, where actual demand is 9225 TWh per year. These values are so close that they hint at a scaling issue, especially considering that other predictions such as mineral extraction levels and emissions are quite close to what we observe. As a side effect, this causes obvious problems with interpretation and comparison to the GHKT model. Unfortunately, due to time constraints, these issues will be resolved in a later stage. It is still very much a work in progress.

A second point regarding calibration is that the model gets stuck in a kind of ”capital trap”, where the low initial capacity of capital to harness energy results in a decline in output, which reinforces a decline in capital until it stabilizes. With capital being the limiting factor, and without increases in the energy-throughput of capital, no growth is possible. Correctly calibrating initial capital is difficult as there are two unknowns: the average energy throughput of capital, and the actual amount of capital in the economy. Keen et al. (2019) note this issue, but they circumvent it by looking at the product of the two, $E_K * K$, as the actual amount of energy used in industry - which can be measured. They then take the function to be expressed only in terms of (usable) energy. In the GHKT model, however, output is expressed in monetary terms, used to derive the initial capital stock, consumption, and ultimately welfare. The approach of looking at industrial energy use therefore does not translate directly to the current model.

Whether or not capital and its ability to harness energy is the limiting factor to production is another interesting point for debate. Despite the known concerns around the energy supply, the amount of proven oil reserves is steadily increasing each year (Ritchie et al., 2023). In terms of coal use, concerns relate more to its emissions than its scarcity as well. Furthermore, the large amounts of subsidies mean that the energy market is not competitive, and energy is not produced at marginal social value. This could mean that in our current economy, energy supply constraints are not the limiting factor to production, as energy is still relatively cheap compared to capital inputs. As theory suggests, when energy is relatively cheap, innovation is mainly capital-directed. The observation that energy consumption is still growing by 2% each year could support the notion that, at a global level, innovations increasing the energy throughput of capital outweigh the energy savings from energy-directed technological change. The findings of this thesis support the idea that economic growth is not (yet) limited by energy constraints, but that it is energy-biased.

A final potential avenue for growth, which is not explored in the model, relates to how economic output is transformed into economic value, represented by parameter η. The current model assumes that additional GDP requires additional usable energy, and that the conversion of usable energy to dollars is constant. The counterargument goes that it may be possible for a given amount of usable energy to generate more economic value, without increasing material or energy throughput. Proponents of this idea often point to digitization or the knowledge economy, where value is added while material throughput is reduced, but as discussed before, there is little evidence that this is happening. A theoretical possibility remains that a shift in preferences could increase the willingness to pay for more "sustainable" products, increasing the value generated per unit of usable energy without violating thermodynamic constraints. It would be a kind of Energy Factor Productivity - producing more output with the same energy inputs. This would indeed change the relationship between energy and the economy. However, it seems unlikely that this will happen in the foreseeable future at a scale large enough to change current implications. Nevertheless, future work could include changes to the conversion parameter. η, to better reflect the dynamics of relative and absolute decoupling.

There are other essential parameters that were not examined in this thesis, most notably the parameter for the elasticity of substitution in energy production, ρ. The impact of this parameter on the energy transition and optimal policy is well researched and deserves a special mention. In the model by Acemoglu et al. (2012) the required level of carbon taxation decreases when clean and dirty energy are easily substitutable, since the transition towards low-carbon technologies can occur with lower relative price adjustments. Similarly,Golosov et al. (2014) also mention that when the elasticity of substitution between fuels is high, the optimal tax induces a faster phase-out of fossil energy and even a decline in global temperature over time.

It would be interesting to expand on the impact of changing these parameters, as well as to include other energy sources that may play an important role in the energy transition, such as natural gas, nuclear energy, and green hydrogen. However, changing the parameter of substitution is not expected to change the trajectory of economic growth when the initial calibration is capital-constrained, as the issues above remain. What could change this trajectory would be a different specification of the production function. For instance, by moving the energy-to-exergy efficiency term to the capital side of the complementary relationship, or by using a specification more like Keen et al. (2019) but with minimal substitution, which could mitigate the current impacts of the capital trap. These are all interesting avenues for further research.

Discussion

Beyond the above limitations of the current model, there are broader questions about how we model the interplay between energy, the economy, and the environment. Both GHKT and DICE are key environmental-economic frameworks in the determination of the optimal tax on emissions. In the wider policy community, the carbon tax is seen as the most efficient method of curbing climate change, but it is also criticized (Bergh & Botzen, 2024). Golosov et al. (2014) comment on this in their paper, stating that in principle, there is no difference between a tax and quantity measures as long as they are allowed to vary over time and across different contexts. They prefer the use of an optimal tax as it does not require any specific knowledge about available stocks of fossil fuels. It is a commonly used argument that to determine efficient quantity measures, one needs to know the exact availability of resource stocks, which is difficult because the economic viability of these stocks can change over time, depending on technologies that may or may not yet exist. However, the optimal tax formula by Golosov et al. (2014) depends on three parameters: the discount rate, the expected damage elasticity, and the depreciation of carbon in the atmosphere. One can ask which of these we are more likely to approximate correctly: the amount of viable resource stocks or the discount rate and the sensitivity of the economy to climate change.

The initial oil stock estimate is also a key determinant of their model outcomes in terms of fossil fuel extraction, carbon emissions, and ultimately economic growth. Although this does not impact the optimal tax rate, it seems resource quantity estimates are already indispensable. Furthermore, GHKT makes very optimistic assumptions on technology in the energy sectors: there is an infinite mineral supply, coal emissions approach zero, and labour productivity grows by 2% per year.[18] In their optimal scenario, with no TFP growth, GHKT

[18] It is difficult to derive what their optimal path for low-carbon energy would translate to in terms of energy or mineral use, as low-carbon energy is only expressed in terms of gigatonnes of carbon, and calibrated based on the relative price between coal and oil. Their low-carbon energy use increases with 452% by 2100, 3478% by 2200, and more than 21 000% by 2300.

predicts global warming of 3.4 °C above preindustrial levels and that GDP levels off around 2050 onward, supporting the idea that without technological improvements, there are limits to growth. This central role of technology has been extensively outlined in this thesis, and linked to increases in usable energy, but some mention must be made of those technologies that could - in theory - delink growth from energy.

It is possible that in the (relatively) near future we can begin the large-scale capturing and storage of carbon, we replace lithium in batteries with salt, we orbit sun-reflecting satellites, or we bring back the Woolly Mammoth which could, and I quote, “Increase resilience of habitats to climate change and environmental upheaval.” We simply do not know. It is important to be open to the potential of human ideas. Daniel Susskind (2024) rightly points out in his book, *Growth: A History and a Reckoning*, that tangible resources might be finite but that “... what really matters for growth are the intangible ideas for combining these resources in new and valuable ways.” This signifies the main point of contention between those who think innovation will allow us to grow indefinitely and those who think it will not.

On the other side of the argument, there are innovation-skeptics, such as Ayres and Warr (2005) and Keen et al. (2019), who believe that what matters for growth is ideas insofar as they enable us to transform energy in new and valuable ways. Smil (2025) similarly cautions that the physical fundamentals of modern civilization have not seen radical changes in the last 50 years. As of today, fossil fuels still supply more than than 80% of our primary energy, and vast amounts of raw materials are needed to capture the energy from sunlight, wind, and water (Smil, 2017). In short, the world runs on ideas once these ideas are *materialized.*

Many innovations often assumed to reduce material requirements might not be so resource-independent. In the case of carbon capturing, this will likely require large industrial facilities with capture equipment, a network of pipelines or ships for transportation, and specialized geological sites for storage. This is unlikely to reduce our mineral and material requirements. That does not exclude the future possibility that technological advancements will allow us to reduce our overall need for energy - sand replacing lithium would be a great start - but waiting for future technological developments risks overlooking the solutions that we already have. In the words of Donella H. Meadows et al. (1972), “Is it preferable to go on growing until some other natural limit arises, in the hope that at that time another technological leap will allow growth to continue still longer?” This brings me to the final point: How do we move forward?

It has been more than 50 years since the first publication of *The Limits to Growth* and more than a decade since the Paris climate agreement, but worldwide energy demand and fossil fuel usage have continued to increase. Even if stated climate policies are met, these are currently not enough to limit global temperature increases to below 1.5 °C, nor are they expected to lead to a decline in our energy and material use. A carbon tax alone may therefore not be enough or not suited for mitigating challenges around energy and resource constraints. It is beyond the scope of this thesis to discuss them at length, but there is a range of policy options that address the physical basis of the economy more directly, from supply-side measures, such as caps on fossil fuel extraction, or demand-side policies, by enabling longer product life-spans or targeting value shifts. However, if usable energy and economic growth are as closely linked as the findings of this thesis suggest, any meaningful reduction in energy throughput may come at the expense of economic growth.

Taking a step back, the question is not only which policies to pursue, but to what end. If the aim is to live well within the planetary boundaries with which this thesis began, this might require modeling them as hard constraints, non-negotiables, rather than as outcome variables when deciding on the optimal way forward. Applying a strict carbon, energy, or material budget to commonly used economic models also presents an interesting avenue for future research.

10 Conclusion

This thesis adapts an existing DSGE framework by Golosov et al. (2014), GHKT, to illustrate the impact of energy constraints on the energy transition and economic growth. Two important changes have been made. Drawing on the work of Keen et al. (2019) the model uses an energy-based nested Cobb-Douglas production function, where energy and the capacity of capital to use energy become perfect complements. This better aligns with physical laws to signify that without energy, there is no production. A second important change is the introduction of mineral constraints to the production of low-carbon energy, largely inspired by Chazel et al. (2023). Unlike many commonly used integrated assessment models, this imposes a biophysical constraint on low-carbon energy expansion, recognizing that the sector cannot grow indefinitely at no environmental cost. Both these changes to the model have strong implications for the low-carbon and final goods sector, varying with different assumptions on technological innovation.

The results show that mineral constraints matter for the scale and timing of the energy transition. The exact impact of scarcity depends on technological assumptions, but in all specifications with mineral constraints, low-carbon energy will eventually reach a plateau. The level of this plateau is higher when primary minerals are used more efficiently, when green capital depreciates more slowly, or when the primary mineral stock is larger. However, if the mineral stock is sufficiently small, technological innovation no longer offsets the impact of increasing scarcity. In line with Chazel et al. (2023), Fabre et al. (2020), and Pommeret et al. (2022), a larger mineral stock enables a higher share of low-carbon energy in the energy mix, reducing the importance of oil and coal, and subsequently lowering emissions compared to the baseline scenario without changes to technology. Yet in contrast to GHKT, fossil fuels remain relatively more important in the overall energy mix, as low-carbon energy cannot grow indefinitely.

Where the introduction of mineral constraints mainly changes trajectories in the energy transition, the new production function has strong implications for how economic growth emerges. Rather than stemming from an exogenous increase in total factor productivity, economic growth is linked to biophysical increases in usable energy - exergy. The amount of usable energy that is available to produce output is determined by the complementary relationship between the usable energy supply and the ability of capital to harness that energy. Growth then happens through capital-directed innovation, enabling a higher output per unit of capital, or through energy-directed innovation, lowering the energy intensity per unit of output. Innovations in energy efficiency reduce overall energy demand, but capital-direct innovation has an energy bias, meaning it increases energy use. Whether energy constraints can be relieved thus depends on the balance between energy-saving and energy-expanding innovations.

In the current model, given the complementary relationship between energy and capital, capital is the limiting factor to production. In this capital-constrained economy, output can only expand if capital's ability to harness energy increases. This inevitably comes with a higher energy demand. As low-carbon energy is constrained by the scarcity of minerals, this demand is then met by the increased use of fossil fuels, raising emissions and temperature as a consequence. Without this increase in usable energy through capital, there is no economic growth. Although the initial calibration of a "capital-constrained" economy can be contested, the finding that additional growth is mainly driven by increases in usable energy is supported by empirical observations, where global energy and fossil fuel demand continue to rise despite massive gains in energy and carbon efficiency.

Relative to Golosov et al. (2014), these changes lead to a markedly different trajectory compared to their benchmark. Where they find that even in optimum, temperature rises by 3.4 °C, the current model predicts temperature increases well below 2 °C - with and without a carbon tax. The reconfiguration of the production function together with the additional constraints to low-carbon energy production impose a kind of self-regulating mechanism, where growth is constrained by energy, and energy production is constrained by scarcity, carbon emissions, or both. The carbon tax is then less necessary as a corrective tool, as even without it, GDP will plateau at a lower level and the associated cumulative emissions will decline before global warming reaches critical levels.

Overall, these findings support the idea that energy should be taken seriously when modeling the energy transition and economic growth. Optimistic assumptions on the prospects of innovation may not hold in the case of the energy transition, and focusing on emissions alone may overlook the impact of diminishing mineral reserves. In principle, scarcity can be alleviated by technological progress, but only if the speed and direction of innovation exceeds the combined impact of increasing energy demand and increasing mineral scarcity. In other words, not all innovation matters equally. Where some innovations contribute to growth through biophysical increases in usable energy, others might reduce the overall energy intensity while not directly contributing to increases in output or GDP. This requires a shift in our economic thinking and modeling practices. To achieve true sustainability, the challenge ahead is not only technical but conceptual: to build an economy that thrives without needing to grow beyond the planet that sustains it.

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A Appendix

1. Chazel et al. (2023): Green capital production

Chazel et al. (2023) model green capital to be produced by a combination of inputs of primary minerals, m_p, and secondary minerals, m_s. Primary mineral extraction, $m_{p,t,}$, is modeled like coal in GHKT, where it requires labour $L_{p,t,}$ whose productivity is $A_{p,t}$. Primary mineral extraction is bounded by the remaining primary mineral reserves, $M_{p,t}$:

$$m_{p,t} = A_{p,t} L_{p,t}$$

$$m_{p,t} \leq M_{p,t}$$

$$M_{p,t+1} = M_{p,t} - m_{p,t}$$

Secondary mineral extraction also requires labour, $L_{s,t}$ whose productivity is denoted by $A_{s,t,}$. And it is bounded by the total available reserve of secondary mineral, $M_{s,t}$:

$$m_{s,t} = A_{s,t} L_{s,t}$$

$$0 \leq m_{s,t} \leq M_{s,t}$$

As green capital depreciates entirely over the course of one period (see below) the secondary mineral stock is built up accordingly like:

$$M_{s,t+1} = M_{s,t} - m_{s,t} + (m_{s,t} + m_{p,t})$$

which gives

$$M_{s,t+1} = M_{s,t} + m_{p,t}$$

Through this, they can model the impact of recycling on the timing of the peak and plateau of low-carbon energy production.

For this, they first model the production of green capital, where κ_s and κ_p are the share parameters of primary and secondary minerals, and the parameter of substitution, $\bar{\rho}$, determines how easily primary and secondary minerals can be substituted for each other in the production of green capital:

$$G_t = (\kappa_s m_{s,t}^{\bar{\rho}} + \kappa_m m_{p,t}^{\bar{\rho}})^{\frac{1}{\bar{\rho}}}$$

Lastly, green capital produced in period t then contributes to the production of low-carbon energy in that period, $E_{3,t}$, alongside labour. The output from labour in the low-carbon sector in turn depends on its productivity $A_{3,t}$ and the size of the labour force dedicated to the sector $L_{3,t}$. The parameters κ_s and κ_p again indicate the relative shares of each input in the production of low-carbon energy, and $\ddot{\rho}$ indicates the parameter of substitution between inputs.

$$E_{3,t} = [\kappa_L (A_{3,t} L_{3,t})^{\ddot{\rho}} + \kappa_G (\psi G_t)^{\ddot{\rho}}]^{\frac{1}{\ddot{\rho}}}$$

The calibration tables below illustrate how they parameterized their model and which changes were made.

Additional tables

Table 2: Changes to parameters under different scenarios

Parameter	Baseline	S1	S2	S3	S4	S5	S6	S7	S8
$\delta\kappa_M$	-	0.002	-	0.002	0.002	-	-	-	0.002
δ_G	1	1	0.1	0.1	0.1	-	-	-	0.1
M_0	2000	2000	2000	2000	[500, 1000]	2000	2000	2000	2000
δe_x	-	-	-	-	-	0.02	-	0.02	0.02
δE_K	-	-	-	-	-	-	0.02	0.02	0.02

Table 3: Calibration parameters from GHKT and Chazel

Parameter	Interpretation	Value	Unit	Source
β	discount rate	0.985	annual	GHKT
Δ	depreciation rate of capital	1	% per decade	GHKT
g_{A3}	productivity growth low-carbon sector	0.02	% per year	GHKT
L	labour supply	1	-	GHKT
$1 - d_s$	amount of carbon in the atmosphere that does not decay	$\phi_L + (1 - \phi_L)\phi_0(1 - \phi)^S$	-	GHKT
ϕ	rate of carbon depreciation	0.0228	% annual	GHKT
ϕ_L	share of carbon staying in the atmosphere forever	0.2	% annual	GHKT
ϕ_0	remaining carbon that exits the atmosphere within a decade	0.393	% annual	GHKT
γ	damage elasticity	0.000023793	-	GHKT
ρ	parameter of substitution in the production of energy	-0.058	-	GHKT
$\ddot{\rho}$	parameter of substitution in the green capital production function	-3	-	Chazel
M_0	Initial mineral stock	2000	$MtCu$	Chazel

Table 4: Calibration parameters of the current model

Parameter	Interpretation	Baseline Value	Unit
α	output elasticity of capital	0.33	-
δ_G	depreciation rate of green capital	1	% annual
ψ	energy produced (over one period) per unit of green capital	1.462	$\frac{x1000TWh}{MtCu}$
κ_1	share parameter of oil in the production of E_3	0.455	-
κ_2	share parameter of coal in the production of E_3	0.078	-
κ_3	share parameter of low-carbon energy in the production of E_3	0.467	-
$A_{2,0}$	labour productivity in the coal sector	11,169	x1000 TWh/decade
$A_{3,0}$	labour productivity in the low-carbon energy sector	3399	x1000 TWh/decade
R_0	initial oil stock	2720	x1000 TWh
M_0	initial mineral stock	2000	$MtCu$
$\kappa_{M,t}$	share parameter of minerals in the production of green capital	0.75	-
$\kappa_{L,t}$	share parameter of labour in the production of green capital	0.25	-
$e_{x,0}$	initial energy-to-exergy efficiency	0.33	-
E_0	initial energy supply	4.17	x1000 TWh/decade
$E_{K,t}$	initial usable energy throughput of capital	$\frac{e_{x,0}E_0}{K_0}$	x1000 TWh/billion dollars
$e_{x,0}$	initial energy-to-exergy efficiency	0.33	-
Y_{2024}	initial output	110 000	billion US dollars
η	conversion of usable energy to GDP	$\frac{Y_0}{Y_{2024}}$	x1000TWh/billion dollars

Additional figures

The emission coefficient of coal

Taken from Golosov et al. (2014), the emissions coefficient, ypsilon, determines the emission intensity of coal over time, assuming a sharp decline in emissions from 2150 onwards, meaning coal eventually emerges as a clean backstop technology.

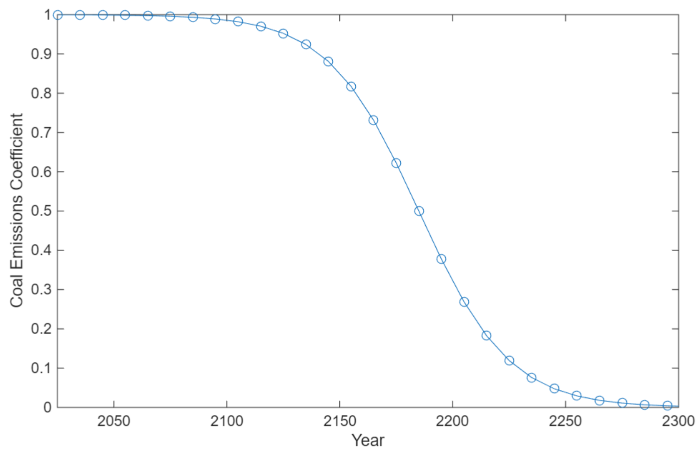

Figure A.1: Emission coefficient, Υ

Oil use

As can be seen, oil use and its associated emissions are constant, and differ little between the optimal and the laissez-faire scenario, similar to GHKT.

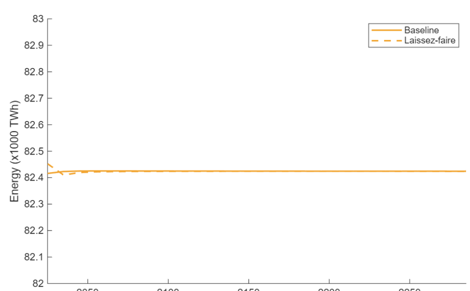

Figure A.2: Oil use: optimum versus laissez-faire

Coal emissions

When plotting emissions from coal against Golosov et al. (2014), it becomes clear why their temperature rises more than the current model.

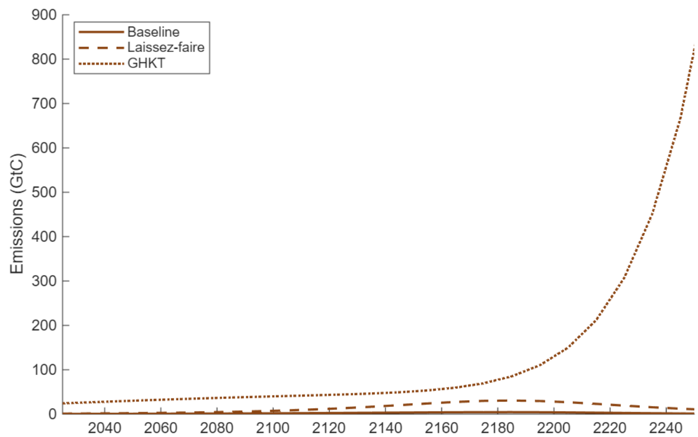

Figure A.3: Emissions from coal production

Low-carbon energy labour share GHKT

To achieve high growth of low-carbon energy, the labour share going to the sector starts at around 2%, which then declines when labour productivity increases over time.

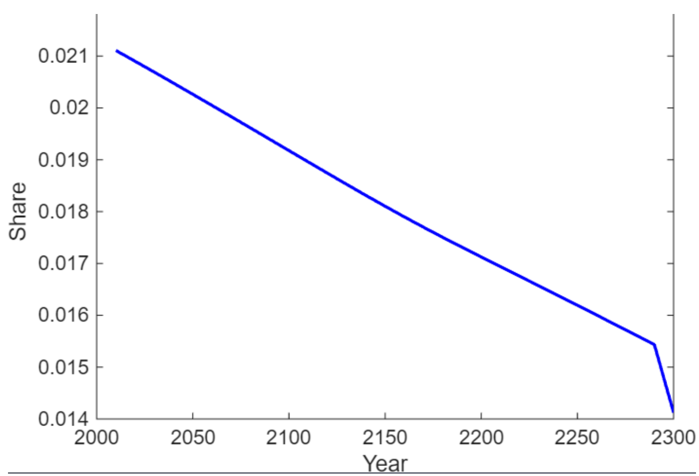

Figure A.4: Labour share E3, GHKT

The energy supply under different technologies

Where innovation in the relative efficiency and depreciation of minerals increases the energy supply, this effect is no longer visible when the mineral stock is reduced.

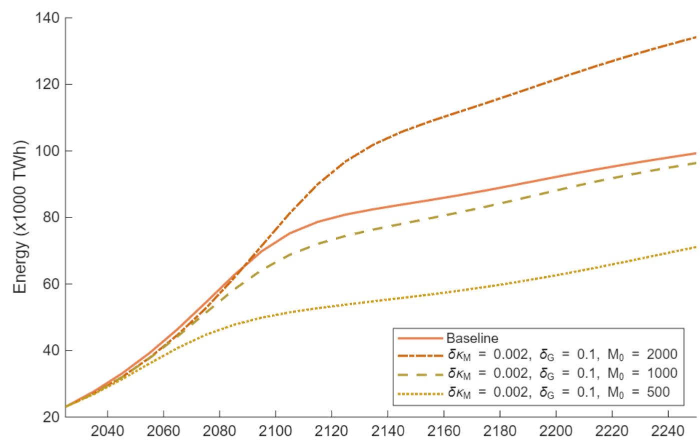

Figure A.5: Total energy

Absolute GDP

As can be seen, GDP in GHKT also reaches a plateau (as their baseline assumes no TFP growth) yet at a higher level than in the current model.

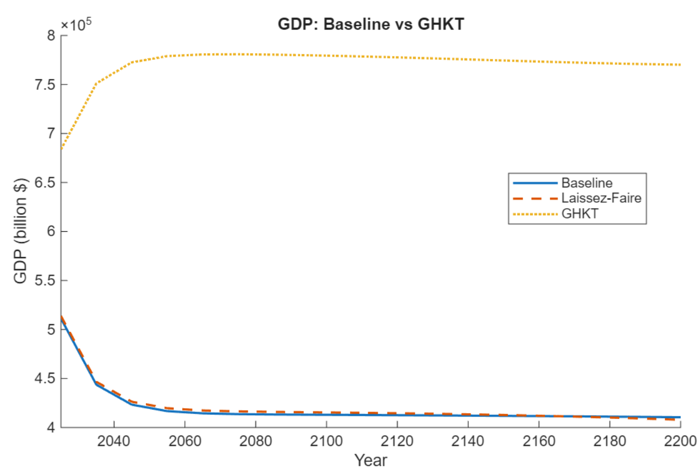

Figure A.6: GDP: baseline, LF & GHKT

The energy-capital ratio

The energy-capacity versus energy-supply ratio highlights that the model is "capital constrained", resulting in the drop in output and stagnating GDP.

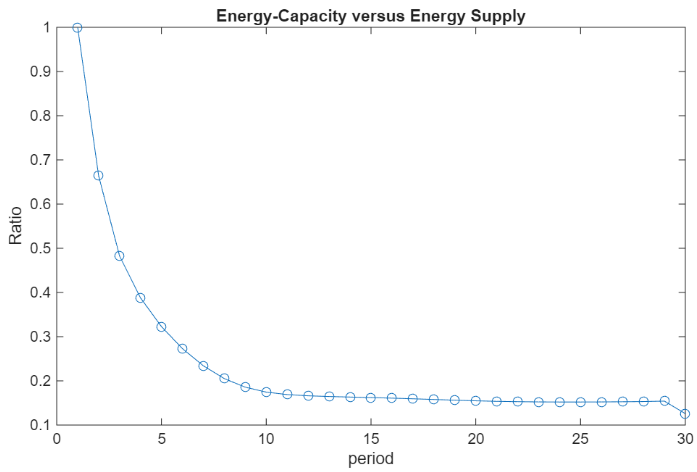

Figure A.7: Baseline $\frac{E_K * K}{e_X * E}$

Statement on the use of AI

The relatively recent introduction of generative AI tools has definitely changed the way we study and the way we learn. I cannot deny that AI has been a complement to my work, and that it has overall contributed to a better thesis experience.

Around 90% of my use of AI related to understanding code better, and for this I used Gemini and ChatGPT. Critical thinking is of course always required. I learned the hard way (after attempting to translate MATLAB code into Python) that AI is is very good at pretending to be correct. Still, it sped up my learning process because rather than scrolling through StackOverflow or Reddit for hours, I can input my exact code and receive an answer immediately. Not only will it then tell me what the error might relate to, but I can ask additional questions about where I went wrong in my thinking process. This helps me retain the information better. I basically used AI like an advanced coding cheat sheet. Additionally, I used Consensus in the proposal phase of my thesis to find relevant literature, and I used the built-in Copilot feature in the MATLAB app for debugging.

That being said, I want to state clearly that I never used AI to generate original content and that everything in this thesis was written by me alone, word for word. I thus take full responsibility for the content of this thesis.